Received: *October 9, 1998,* Accepted: *June 15, 1999*

On gauged maximal supergravity in six dimensions

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Abstract: The maximal SO(5) gauged D=7 supergravity is dimensionally reduced to six dimensions yielding a new SO(5) gauged D=6 model. It is shown that, unlike in D=7, the SO(5) gauge coupling constant can be taken to zero to yield the maximally extended supergravity in six dimensions. It is also shown that the limit of D=5 N=4 SU(2)×U(1) gauged supergravity in which the U(1) coupling constant is turned off can be obtained.

Keywords: Gauge Symmetry, Extended Supersymmetry, Supergravity Models.

JHEP06(1999)018

Contents

1. Introduction

Due to the recently conjectured AdS/CFT correspondence [1] and its generalisation, the domain-wall/QFT correspondence [2], the determination of all possible gaugings of higher dimensional supergravities is currently attracting much interest. Gauged maximal supergravities are known to exist in dimensions $4, 5, 7$ and 8, see refs [3]–[10]. There is a clear lack of a gauged maximal supergravity in six dimensions, thus the presentation of such a model will be the focus of this paper.

To understand the construction of a gauged maximal supergravity in D=6 we begin with the ungauged model of Tanii [11] for which the bosonic field content is 1 graviton, 5 second rank antisymmetric tensor potentials, 16 vectors and 25 scalars. The model has (4,4) maximal supersymmetry i.e. 4 spinor charges of each chirality, and the automorphism group[1] of the superalgebra is SO(5)×SO(5). The scalars parametrise the coset space SO(5,5)/SO(5)×SO(5). The 16 vector fields are in the $\underline{16}$ spinor representation of SO(5,5) which decomposes into the (4,4) representation of SO(5)×SO(5). Clearly there are not enough vectors to gauge SO(5)×SO(5). However, the (4,4) representation of SO(5)×SO(5) decomposes w.r.t. its diagonal SO(5) subgroup in the following way

$$(4,4) = \underline{10} + \underline{5} + \underline{1}\,. \tag{1.1}$$

[1] Usp(4)≃Spin(5), the covering group of SO(5).

The $\underline{10}$ could be used to gauge SO(5), but there are potential consistency problems with the $\underline{5}$; one may try to solve these problems using constructions similar to those in D=7 and D=5 [12], but these only work in odd dimensions. Alternatively one could try to gauge the larger group ISO(5) for which the $\underline{10} + \underline{5}$ of SO(5) is the adjoint $\underline{15}$. There also remains the question of what the ground state of this D=6 gauged model will be; due to the lack of a maximally extended supergroup in six dimensions, there cannot be a maximally supersymmetric AdS_6 vacuum.

The solution to the gauging of Tanii's model is provided by the S^1 reduction of the D=7 SO(5) gauged supergravity [8]. The bosonic field content is 1 graviton, 5 third rank massive self dual antisymmetric tensor potentials, 10 vectors and 14 scalars. One feature of D=7 SO(5) gauged supergravity that is relevant is that the scalar potential has a supersymmetry preserving maximum, leading to an AdS_7 vacuum with isometry supergroup OSp(6,2|4) [9]. Thus there is no $M_6 \times S^1$ vacuum solution and so dimensional reduction of this model was thought to be inconsistent. In fact as was pointed out in [13] the reduction is perfectly consistent and is implemented using the standard Kaluza-Klein (KK) ansatz on the fields; substitution of the standard KK ansatz for the fields into the lagrangean gives a lower dimensional lagrangean whose field equations are the same as those obtained by direct substitution of the KK ansatz into the higher dimensional field equations. Having said that the reduction is always consistent, one is not guaranteed solutions in the lower dimension unless there exists a solution with a U(1) isometry in the higher dimension. In the case of D=7 maximal SO(5) gauged supergravity, the AdS_7 ground state has a U(1) isometry and, as shown in [13], upon reduction becomes a D=6 domain wall preserving 1/2 of the supersymmetry. Hence, there is no obstacle to the S^1 reduction of D=7 SO(5) gauged supergravity.

Proceeding with the reduction it is clear that the gauge group in D=6 will also be SO(5). The 10-plet of vectors survives in D=6 and is supplemented by a single KK vector. Hence there are 11 vectors in D=6 transforming under SO(5) as a $\underline{10}+\underline{1}$. The 5-plet of third rank massive self dual antisymmetric tensor potentials splits into a 5-plet of massive 3-index potentials and a 5-plet of massive 2-index potentials. As the self duality is lost in the reduction, the 3-index potentials are auxiliary and can be eliminated. Hence in D=6 the massive 2-index tensor potentials transform as a $\underline{5}$; this accounts for the remaining 5 vectors and massless 2-index potentials of the ungauged theory. Thus, understanding the SO(5) gauged D=6 model is much easier from the D=7 perspective, as is its construction.

Another interesting feature of D=7 SO(5) gauged supergravity is that the global limit, in which the SO(5) gauge coupling constant is turned off, cannot be obtained. One might suspect that this will also be true in D=6. We find in fact that this is not the case and the global limit can be taken smoothly allowing us to make contact with the maximal D=6 supergravity [11].

The paper is set out as follows. In section 2 we perform the dimensional reduction of the bosonic sector of D=7 SO(5) gauged supergravity to six dimensions and obtain a new D=6 SO(5) gauged model. In section 3 we show the $g \to 0$ limit is attainable after appropriate rescalings. In section 4 we present our conclusions. In appendices A and C we give the complete lagrangeans and supersymmetry transformation laws of D=6 SO(5) supergravity and D=6 maximal supergravity respectively. On the subject of taking limits, using a similar method, we also show in appendix D that the limit of D=5 N=4 SU(2)×U(1) gauged supergravity in which the U(1) coupling constant is turned off can be obtained.

2. Reduction to D=6

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First we recall some features of the construction of maximal SO(5) gauged D=7 supergravity. One of these is the necessity for 'odd dimensional self duality' [8, 12]. In the ungauged maximal D=7 model [14] there are five third rank antisymmetric tensor potentials $A_{\alpha\beta\gamma I}$. These fields are expected to transform in the vector representation of SO(5) upon gauging but simply replacing the ordinary derivative by an SO(5) covariant derivative in the Maxwell action of $A_{\alpha\beta\gamma I}$ would break the antisymmetric tensor gauge invariance. Thus there would not be a matching of bosonic and fermionic degrees of freedom necessary for supersymmetry. A way around this problem is provided by 'odd dimensional self duality'. In seven dimensions a massless 3-index antisymmetric tensor has the same number of degrees of freedom as a massive self dual 3-index antisymmetric tensor, namely ten. Therefore the Maxwell action of the massless $A_{\alpha\beta\gamma I}$ fields can be replaced by an action whose field equation reads

$$mS_3 = \epsilon^7 dS_3 \,, \tag{2.1}$$

hence the name self duality in odd dimensions. Iteration of this equation yields a massive Proca equation. Therefore the 3-index fields have only ten propagating degrees of freedom. However now one is in a position to effect the gauging as the ordinary derivative can simply be replaced by an SO(5) covariant derivative. The antisymmetric tensor gauge invariance no longer exists but isn't needed as the potentials $S_{\alpha\beta\gamma I}$ each propagate the correct number of degrees of freedom required by supersymmetry and also transform as a 5-plet under the local SO(5) symmetry. The parameter m turns out to be proportional to the SO(5) coupling constant g.

A side-effect of 'odd dimensional self duality' is that there is a 'gauge discontinuity' i.e. g appears non analytically in the Chern-Simons terms and in the supersymmetry transformation law of $S_{\alpha\beta\gamma I}$; hence no $g \to 0$ limit exists. We will see that after reduction to D=6 the $g \to 0$ limit can be taken. We begin with the bosonic sector of D=7 maximal SO(5) gauged supergravity, which is [8][2]

$$
\begin{aligned}
e^{-1}\mathcal{L}_7 = {} & \frac{R}{2} - \frac{1}{4}\left(\Pi_I{}^i \Pi_J{}^j F^{IJ}_{\alpha\beta}\right)^2 - \frac{m^2}{2}\left(\Pi^{-1}{}_i{}^I S_{\alpha\beta\gamma I}\right)^2 + e^{-1}\frac{m}{48}\epsilon^{\alpha\beta\gamma\delta\epsilon\eta\xi}\delta^{IJ} S_{\alpha\beta\gamma I} F_{\delta\epsilon\eta\xi J} - \\
& -\frac{1}{2}P_{\alpha ij}P^{\alpha ij} + \frac{m^2}{4}(T^2 - 2T_{ij}T^{ij}) - \frac{ie^{-1}}{16\sqrt{3}}\epsilon^{\alpha\beta\gamma\delta\epsilon\eta\xi}\epsilon_{IKLMN}\delta^{IJ} S_{\alpha\beta\gamma J} F^{KL}_{\delta\epsilon} F^{MN}_{\eta\xi} + \\
& + \frac{e^{-1}}{8m}\epsilon^{\alpha\beta\gamma\delta\epsilon\eta\xi}\mathrm{Tr}\Big(B_\alpha F_{\beta\gamma} F_{\delta\epsilon} F_{\eta\xi} - \frac{4}{5} g B_\alpha B_\beta B_\gamma F_{\delta\epsilon} F_{\eta\xi} - \frac{2}{5} g B_\alpha B_\beta F_{\gamma\delta} B_\epsilon F_{\eta\xi} + \\
& \qquad + \frac{4}{5} g^2 B_\alpha B_\beta B_\gamma B_\delta B_\epsilon F_{\eta\xi} - \frac{8}{35} g^3 B_\alpha B_\beta B_\gamma B_\delta B_\epsilon B_\eta B_\xi \Big) - \\
& - \frac{e^{-1}}{16m}\epsilon^{\alpha\beta\gamma\delta\epsilon\eta\xi}\mathrm{Tr}\left(B_\alpha F_{\beta\gamma} - \frac{2}{3} g B_\alpha B_\beta B_\gamma\right)\mathrm{Tr}(F_{\delta\epsilon}F_{\eta\xi}) \, , \qquad (2.2)
\end{aligned}
$$

where the various field strengths are defined in terms of their potentials as follows

$$
\begin{aligned}
P_{\alpha ij} &= \Pi^{-1}{}_{(i}{}^I \left(\delta_I{}^J \partial_\alpha + 2m B_{\alpha I}{}^J\right) \Pi_J{}^k \delta_{j)k} \\
T_{ij} &= \Pi^{-1}{}_i{}^I \Pi^{-1}{}_j{}^J \delta_{IJ} \qquad\qquad T = T_{ij}\delta^{ij} \\
F_{\alpha\beta}{}^{IJ} &= \delta^{IK} F_{\alpha\beta K}{}^J \\
F_{\alpha\beta I}{}^J &= 2\left(\partial_{[\alpha} B_{\beta]I}{}^J + g B_{[\alpha I}{}^K B_{\beta]K}{}^J\right) \\
F_{\alpha\beta\gamma\delta I} &= 4\left(\partial_{[\alpha} S_{\beta\gamma\delta]I} + g B_{[\alpha I}{}^J S_{\beta\gamma\delta]J}\right) . \qquad (2.3)
\end{aligned}
$$

The parameter m is given in terms of the SO(5) coupling constant g as $g = 2m$. $I, J, \ldots = 1, \ldots, 5$ are SO(5) vector indices and $i, j, \ldots = 1, \ldots, 5$ are also vector indices but of a different local composite $\mathrm{SO}(5)_c$ group whose origin lies in the local Lorentz group in eleven dimensions. The scalars $\Pi_I{}^i$ parametrise the coset space $\mathrm{SL(5,R)}/\mathrm{SO}(5)_c$. $B_\alpha{}^{IJ} = -B_\alpha{}^{JI}$ and we use the mostly plus metric convention.

To perform the reduction to D=6 the ansatz for the fields are:

$$
\hat{e}\,{}^{\hat{a}}_{\hat{\alpha}} = \begin{pmatrix} e^{\frac{\sigma}{2\sqrt{10}}} e\,{}^{\underline{\mu}}_{\mu} & 0 \\ e^{-\frac{2\sigma}{\sqrt{10}}} A_\mu & e^{-\frac{2\sigma}{\sqrt{10}}} \end{pmatrix} , \qquad (2.4)
$$

where hats refer to D=7, $\hat{a} = (\underline{\mu}, \underline{z})$ are local Lorenz indices and $\hat{\alpha} = (\mu, z)$ are world indices.

$$
d\hat{S}_7^2 = e^{\frac{\sigma}{\sqrt{10}}} dS_6^2 + e^{-\frac{4\sigma}{\sqrt{10}}}(dz + A)^2 \, , \qquad (2.5)
$$

where $A = A_\mu dx^\mu$ and $f = dA$.

$$
\begin{aligned}
\hat{\Pi}_I{}^i(x^\mu, z) &= \Pi_I{}^i(x^\mu) \\
\hat{B}_{1I}{}^J &= B_{1I}{}^J + B_{0I}{}^J(dz + A) \\
\hat{S}_{3I}{}^J &= S_{3I}{}^J + S_{2I}{}^J(dz + A) \, . \qquad (2.6)
\end{aligned}
$$

[2]We note that the sign of the mass term of the 3-index field is opposite to the one in [8]. That this is the correct sign can be checked by choosing $\mathrm{S}_{123} = u$, $\mathrm{S}_{456} = v$, where u and v depend only on time, and letting all other components of $\mathrm{S}_{\alpha\beta\gamma}$ vanish. With the choice $\Pi^{-1}{}_j{}^I = \delta^I_j$, the lagrangean (neglecting interaction terms) for the 3-index field then becomes $\frac{\dot{v}}{2}^2 - \frac{m}{2}^2 v^2$ after elimination of the auxiliary field u. Hence has the required form KE−PE.

The resulting six dimensional lagrangean is:

$$\begin{aligned}
e^{-1}\mathcal{L}_6 = &\frac{R}{2} - \frac{1}{8}e^{-\frac{5\sigma}{\sqrt{10}}}(f_{\mu\nu})^2 - \frac{1}{4}e^{-\frac{\sigma}{\sqrt{10}}}\left(\Pi_I{}^i\Pi_J{}^jF^{IJ}_{\mu\nu}\right)^2 - \frac{m^2}{2}e^{-\frac{2\sigma}{\sqrt{10}}}\left(\Pi^{-1}{}_i{}^IS_{\mu\nu\rho I}\right)^2 - \\
&-\frac{3m^2}{2}e^{\frac{3\sigma}{\sqrt{10}}}\left(\Pi^{-1}{}_i{}^IS_{\mu\nu I}\right)^2 - \frac{1}{2}P_{\mu ij}P^{\mu ij} - \frac{1}{2}e^{\frac{4\sigma}{\sqrt{10}}}\left(\Pi_I{}^i\Pi_J{}^jF^{IJ}_{\mu}\right)^2 - \frac{1}{4}(\partial_\mu\sigma)^2 - \\
&-\frac{m^2}{2}e^{\frac{5\sigma}{\sqrt{10}}}\left(\Pi^{-1}{}_{(i}{}^IB_{0I}{}^J\Pi_J{}^k\delta_{j)k}\right)^2 + \frac{m^2}{4}e^{\frac{\sigma}{\sqrt{10}}}\left(T^2 - 2T_{ij}T^{ij}\right) + \\
&+e^{-1}\frac{m}{12}\delta^{IJ}\epsilon^{\mu\nu\rho\sigma\lambda\tau}\Big[S_{\mu\nu\rho I}G_{\sigma\lambda\tau J} - gS_{\mu\nu\rho I}B_{0J}{}^KS_{\sigma\lambda\tau K} + \\
&\qquad + \frac{3}{4}S_{\mu\nu I}G_{\rho\sigma\lambda\tau J} + \frac{9}{2}S_{\mu\nu I}S_{\rho\sigma J}\partial_\lambda A_\tau\Big] + \frac{e^{-1}}{m}\Omega(B) - \\
&-e^{-1}\frac{\sqrt{3}i}{12}\epsilon_{JKLMN}\delta^{IJ}\epsilon^{\mu\nu\rho\sigma\lambda\tau}\left[\frac{3}{4}S_{\mu\nu I}F^{KL}_{\rho\sigma}F^{MN}_{\lambda\tau} + S_{\mu\nu\rho I}F^{KL}_{\sigma\lambda}F^{MN}_{\tau}\right],
\end{aligned} \tag{2.7}$$

where

$$\begin{aligned}
P_{\mu ij} &= \Pi^{-1}{}_{(i}{}^I\left(\delta_I{}^J\partial_\mu + gB_{\mu I}{}^J\right)\Pi_J{}^k\delta_{j)k} \\
T_{ij} &= \Pi^{-1}{}_i{}^I\Pi^{-1}{}_j{}^J\delta_{IJ}\,, \qquad T = T_{ij}\delta^{ij}\,, \qquad f_2 = dA \\
F_{2I}{}^J &= dB_{1I}{}^J + B_{0I}{}^JdA + gB_{1I}{}^KB_{1K}{}^J \\
F_{1I}{}^J &= dB_{0I}{}^J + g\left(B_{1I}{}^KB_{0K}{}^J - B_{0I}{}^KB_{1K}{}^J\right) \\
G_{3I} &= dS_{2I} + gB_{1I}{}^JS_{2J}\,, \qquad G_{4I} = dS_{3I} + gB_{1I}{}^JS_{3J}
\end{aligned} \tag{2.8}$$

and the Chern-Simons term $\Omega(B)$ is given in appendix B.

After integrating by parts and dropping a total derivative we find

$$\frac{m}{16}\epsilon^{\mu\nu\rho\sigma\lambda\tau}S_{\mu\nu}{}^IG_{\rho\sigma\lambda\tau I} = \frac{m}{12}\epsilon^{\mu\nu\rho\sigma\lambda\tau}S_{\mu\nu\rho}{}^IG_{\sigma\lambda\tau I}\,, \tag{2.9}$$

so the terms in the lagrangean involving S_{2I} and S_{3I} can be compactly written as [12]

$$\begin{aligned}
\mathcal{L}'_6 = &\, m^2S_{\mu\nu I}Q^{\mu\nu\rho\sigma IJ}S_{\rho\sigma J} - m^2S_{\mu\nu\rho I}P^{\mu\nu\rho\sigma\lambda\tau IJ}S_{\sigma\lambda\tau J} - \\
&-\frac{\sqrt{3}i}{12}\epsilon_{JKLMN}\epsilon^{\mu\nu\rho\sigma\lambda\tau}S_{\mu\nu}{}^JF^{KL}_{\rho\sigma}F^{MN}_{\lambda\tau} + \\
&+\frac{m}{6}\delta^{IJ}\epsilon^{\mu\nu\rho\sigma\lambda\tau}S_{\mu\nu\rho I}\left(G_{\sigma\lambda\tau J} - \frac{\sqrt{3}i}{2m}\epsilon_{JKLMN}F^{KL}_{\sigma\lambda}F^{MN}_{\tau}\right),
\end{aligned} \tag{2.10}$$

where the operators P and Q are defined by

$$\begin{aligned}
P^{\mu\nu\rho\sigma\lambda\tau IJ} &= \frac{e}{2}e^{-\frac{2\sigma}{\sqrt{10}}}\Pi^{-1}{}_i{}^I\Pi^{-1\,iJ}g^{\mu\sigma}g^{\nu\lambda}g^{\rho\tau} + \frac{1}{6}\delta^{IK}\epsilon^{\mu\nu\rho\sigma\lambda\tau}B_{0K}{}^J \\
Q^{\mu\nu\rho\sigma IJ} &= -\frac{3e}{2}e^{\frac{3\sigma}{\sqrt{10}}}\Pi^{-1}{}_i{}^I\Pi^{-1\,iJ}g^{\mu\rho}g^{\nu\sigma} + \frac{3}{8m}\delta^{IJ}\epsilon^{\mu\nu\rho\sigma\lambda\tau}\partial_\lambda A_\tau\,.
\end{aligned} \tag{2.11}$$

We have seen that the 3-index antisymmetric tensor potential in D=7 splits into a 2-index and another 3-index tensor on reduction to D=6. The D=7 odd dimensional self duality then allows the elimination of the 3-index tensor in favour of the 2-index tensor. Variation of $\mathcal{L}_6^{'}$ w.r.t. S_{3I} gives the equation

$$P^{\mu\nu\rho\sigma\lambda\tau IJ} S_{\sigma\lambda\tau J} = \frac{1}{12m} \delta^{IJ} \epsilon^{\mu\nu\rho\sigma\lambda\tau} \left(G_{\sigma\lambda\tau J} - \frac{\sqrt{3}i}{2m} \epsilon_{JKLMN} F^{KL}_{\sigma\lambda} F^{MN}_{\tau} \right). \tag{2.12}$$

Defining the inverse of P such that

$$P^{\mu\nu\rho\alpha\beta\gamma IJ} (P^{-1})_{\alpha\beta\gamma\sigma\lambda\tau JK} = \delta^{\mu\nu\rho}_{\sigma\lambda\tau} \delta^{I}_{K} \tag{2.13}$$

and using $P^{\mu\nu\rho\sigma\lambda\tau IJ} = P^{\sigma\lambda\tau\mu\nu\rho JI}$ we have

$$S_{\sigma\lambda\tau P} = \frac{1}{12m} \delta^{IJ} \epsilon^{\alpha\beta\gamma\mu\nu\rho} \left(G_{\mu\nu\rho I} - \frac{\sqrt{3}i}{2m} \epsilon_{IKLMN} F^{KL}_{\mu\nu} F^{MN}_{\rho} \right) P^{-1}_{\alpha\beta\gamma\sigma\lambda\tau JP} . \tag{2.14}$$

Substitution back in $\mathcal{L}_6^{'}$ finally yields the bosonic sector of the SO(5) gauged D=6 model:

$$\begin{aligned} \mathcal{L}_6 = & \frac{eR}{2} - \frac{e}{8} e^{-\frac{5\sigma}{\sqrt{10}}} (f_{\mu\nu})^2 - \frac{e}{4} e^{-\frac{\sigma}{\sqrt{10}}} \left(\Pi_I{}^i \Pi_J{}^j F^{IJ}_{\mu\nu} \right)^2 + m^2 S_{\mu\nu I} Q^{\mu\nu\rho\sigma IJ} S_{\rho\sigma J} + \\ & + \frac{1}{144} \left[\delta^{I'J'} \epsilon^{\alpha\beta\gamma\delta\epsilon\eta} (\tilde{G}_{\delta\epsilon\eta I'}) \right] P^{-1}_{\alpha\beta\gamma\mu\nu\rho J'I} \left[\delta^{IJ} \epsilon^{\mu\nu\rho\sigma\lambda\tau} (\tilde{G}_{\sigma\lambda\tau J}) \right] - \\ & - \frac{e}{2} P_{\mu ij} P^{\mu ij} - \frac{e}{2} e^{\frac{4\sigma}{\sqrt{10}}} \left(\Pi_I{}^i \Pi_J{}^j F^{IJ}_{\mu} \right)^2 - \frac{e}{4} (\partial_\mu \sigma)^2 - \frac{em^2}{2} e^{\frac{5\sigma}{\sqrt{10}}} \left(\Pi^{-1}{}_{(i}{}^I B_{0I}{}^J \Pi_J{}^k \delta_{j)k} \right)^2 + \\ & + \frac{em^2}{4} e^{\frac{\sigma}{\sqrt{10}}} \left(T^2 - 2 T_{ij} T^{ij} \right) - \frac{\sqrt{3}i}{16} \epsilon_{JKLMN} \epsilon^{\mu\nu\rho\sigma\lambda\tau} S_{\mu\nu}{}^J F^{KL}_{\rho\sigma} F^{MN}_{\lambda\tau} + \frac{1}{m} \Omega(B) , \end{aligned} \tag{2.15}$$

where

$$\begin{aligned} P_{\mu ij} &= \Pi^{-1}{}_{(i}{}^I \left(\delta_I{}^J \partial_\mu + g B_{\mu I}{}^J \right) \Pi_J{}^k \delta_{j)k} \\ T_{ij} &= \Pi^{-1}{}_i{}^I \Pi^{-1}{}_j{}^J \delta_{IJ} \quad T = T_{ij} \delta^{ij} \quad f_2 = dA \\ F_{2I}{}^J &= dB_{1I}{}^J + B_{0I}{}^J dA + g B_{1I}{}^K B_{1K}{}^J \\ F_{1I}{}^J &= dB_{0I}{}^J + g \left(B_{1I}{}^K B_{0K}{}^J - B_{0I}{}^K B_{1K}{}^J \right) \\ G_{3I} &= dS_{2I} + g B_{1I}{}^J S_{2J} \\ \tilde{G}_{\mu\nu\rho I} &= G_{\mu\nu\rho I} - \frac{\sqrt{3}i}{2m} \epsilon_{IKLMN} F^{KL}_{\mu\nu} F^{MN}_{\rho} \end{aligned} \tag{2.16}$$

and the operator Q is defined as in (2.11). This model has a bosonic field content of 1 graviton, (14+10+1) scalars, (10+1) vectors and 5 second rank massive antisymmetric tensors. As all local supersymmetries of the action remain unbroken after reduction on S^1, this SO(5) gauged model is guaranteed a supersymmetric extension. In appendix A we present the complete lagrangean and supersymmetry transformation laws of this D=6 SO(5) supergravity.[3]

[3]We note that the transformation law of the auxiliary D=6 field S_{3I} becomes, upon its elimination via the field equations, a consistency condition which we expect to be satisfied upon use of the field equations and supersymmetry transformation laws although we have not shown this. Taking the $g = 0$ limit and performing a dualisation, this consistency condition becomes the supersymmetry transformation law of the field $H_{\mu\nu\rho I}$.

3. The $g \to 0$ limit

In this section we show that the parameter g can be taken to zero smoothly. In order to show this it is necessary to rescale the various fields and to regurgitate five vectors from the five 2-index potentials S_{2I}. Each of these five second rank antisymmetric tensor potentials is massive and so propagates 10 degrees of freedom. On taking the $g \to 0$ limit they become massless and so propagate only 6 degrees of freedom. Hence the extra four degrees of freedom from each S_{2I} must form a massless vector. Therefore $S_{\mu\nu I}$ is replaced everywhere by

$$S_{\mu\nu I} - \frac{1}{m^2} G_{\mu\nu I}\,, \tag{3.1}$$

where $G_{\mu\nu I} = 2\partial_{[\mu} S_{\nu]I}$. This is similar to the procedure used to formally recover the massless N=2a D=10 supergravity from the massive model of Romans [15].

To obtain a lagrangean in a form from which the $g \to 0$ limit can be taken, the following field rescalings are necessary

$$\begin{array}{rclcrcl} B_{1I}{}^J & \longrightarrow & g B_{1I}{}^J & \qquad & B_{0I}{}^J & \longrightarrow & g B_{0I}{}^J \\ \Pi_I{}^i & \longrightarrow & \frac{1}{\sqrt{g}}\, \Pi_I{}^i & & \Pi^{-1}{}_i{}^I & \longrightarrow & \sqrt{g}\, \Pi^{-1}{}_i{}^I \\ S_{2I} & \longrightarrow & \sqrt{g}\, S_{2I} & & S_{1I} & \longrightarrow & \sqrt{g}\, S_{1I}\,. \end{array} \tag{3.2}$$

These rescalings cause slight redefinitions in the field strengths. Notice that the operator P scales with a factor of g. The lagrangean now becomes

$$\begin{aligned} \mathcal{L}_6 = {} & \frac{eR}{2} - \frac{e}{8} e^{-\frac{5\sigma}{\sqrt{10}}} (f_{\mu\nu})^2 - \frac{e}{4} e^{-\frac{\sigma}{\sqrt{10}}} \left(\Pi_I{}^i \Pi_J{}^j F^{IJ}_{\mu\nu}\right)^2 - 6em^4 e^{\frac{3\sigma}{\sqrt{10}}} S_{\mu\nu I} \Pi^{-1}{}_i{}^I \Pi^{-1\, iJ} S^{\mu\nu}{}_J + \\ & + \frac{3m^2}{4} \delta^{IJ} \epsilon^{\mu\nu\rho\sigma\lambda\tau} S_{\mu\nu I} S_{\rho\sigma J} \partial_\lambda A_\tau + 12em^2 e^{\frac{3\sigma}{\sqrt{10}}} S_{\mu\nu I} \Pi^{-1}{}_i{}^I \Pi^{-1\, iJ} G^{\mu\nu}{}_J - \\ & - \frac{3}{2} \delta^{IJ} \epsilon^{\mu\nu\rho\sigma\lambda\tau} S_{\mu\nu I} G_{\rho\sigma J} \partial_\lambda A_\tau - 6ee^{\frac{3\sigma}{\sqrt{10}}} G_{\mu\nu I} \Pi^{-1}{}_i{}^I \Pi^{-1\, iJ} G^{\mu\nu}{}_J + \\ & + \frac{3}{4m^2} \delta^{IJ} \epsilon^{\mu\nu\rho\sigma\lambda\tau} G_{\mu\nu I} G_{\rho\sigma J} \partial_\lambda A_\tau + \\ & + \frac{1}{144} \left[\epsilon^{\alpha\beta\gamma\delta\epsilon\eta} \left(\tilde{G}_{\delta\epsilon\eta}{}^{J'}\right)\right] P^{-1}_{\alpha\beta\gamma\mu\nu\rho J' I} \left[\epsilon^{\mu\nu\rho\sigma\lambda\tau} \left(\tilde{G}_{\sigma\lambda\tau}{}^I\right)\right] - \\ & - \frac{e}{2} P_{\mu ij} P^{\mu ij} - \frac{e}{2} e^{\frac{4\sigma}{\sqrt{10}}} \left(\Pi_I{}^i \Pi_J{}^j F^{IJ}_\mu\right)^2 - \frac{e}{4} (\partial_\mu \sigma)^2 - \\ & - 2em^4 e^{\frac{5\sigma}{\sqrt{10}}} \left(\Pi^{-1}{}_{(i}{}^I B_{0I}{}^J \Pi_J{}^k \delta_{j)k}\right)^2 + em^4 e^{\frac{\sigma}{\sqrt{10}}} \left(T^2 - 2T_{ij}T^{ij}\right) - \\ & - \frac{\sqrt{6} i}{4} m^{\frac{5}{2}} \epsilon_{JKLMN} \epsilon^{\mu\nu\rho\sigma\lambda\tau} S_{\mu\nu}{}^J F^{KL}_{\rho\sigma} F^{MN}_{\lambda\tau} + \\ & + \frac{\sqrt{6} i}{4} m^{\frac{1}{2}} \epsilon_{JKLMN} \epsilon^{\mu\nu\rho\sigma\lambda\tau} G_{\mu\nu}{}^J F^{KL}_{\rho\sigma} F^{MN}_{\lambda\tau} + m^3 \Omega(B)\,, \end{aligned} \tag{3.3}$$

where

$$\begin{aligned}
P_{\mu ij} &= \Pi^{-1}{}_{(i}^{\ I}\left(\delta_I{}^J\partial_\mu + g^2 B_{\mu I}{}^J\right)\Pi_J{}^k\delta_{j)k} \\
F_{2I}{}^J &= dB_{1I}{}^J + B_{0I}{}^J dA + g^2 B_{1I}{}^K B_{1K}^{\ J} \\
F_{1I}{}^J &= dB_{0I}{}^J + g^2\left(B_{1I}{}^K B_{0K}^{\ J} - B_{0I}{}^K B_{1K}^{\ J}\right) \\
G_{3I} &= dS_{2I} + g^2 B_{1I}{}^J S_{2J}\,, \qquad\qquad G_{2I} = dS_{1I} \\
\tilde{G}_{\mu\nu\rho I} &= G_{\mu\nu\rho I} - 2\sqrt{3m} i\epsilon_{IKLMN} F^{KL}_{\mu\nu} F^{MN}_{\rho} - 12 B_{\mu I}{}^J G_{\nu\rho J}
\end{aligned} \tag{3.4}$$

and $\Omega(B)$ is given in appendix B.

Noting that the term with an inverse power of m is a total derivative and so can be dropped, the $g \to 0$ limit can now be recovered

$$\begin{aligned}
\mathcal{L}_6 = \frac{eR}{2} - \frac{e}{8}e^{-\frac{5\sigma}{\sqrt{10}}}(f_{\mu\nu})^2 - \frac{e}{4}e^{-\frac{\sigma}{\sqrt{10}}}\left(\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\mu\nu}\right)^2 - 6ee^{\frac{3\sigma}{\sqrt{10}}}\left(\Pi^{-1}{}_i^{\ I} G_{\mu\nu I}\right)^2 + \\
+\frac{1}{144}\left[\epsilon^{\alpha\beta\gamma\delta\epsilon\eta}(G_{\delta\epsilon\eta}{}^I)\right] P^{-1}_{\alpha\beta\gamma\mu\nu\rho IJ}\left[\epsilon^{\mu\nu\rho\sigma\lambda\tau}(G_{\sigma\lambda\tau}{}^J)\right] - \frac{e}{2}P_{\mu ij}P^{\mu ij} - \\
-\frac{e}{2}e^{\frac{4\sigma}{\sqrt{10}}}\left(\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\mu}\right)^2 - \frac{e}{4}(\partial_\mu\sigma)^2 - \frac{3}{2}\delta^{IJ}\epsilon^{\mu\nu\rho\sigma\lambda\tau} S_{\mu\nu I} G_{\rho\sigma J}\partial_\lambda A_\tau\,,
\end{aligned} \tag{3.5}$$

where

$$\begin{aligned}
f_2 &= dA\,, \qquad\qquad G_{2I} = dS_{1I} \\
P_{\mu ij} &= \Pi^{-1}{}_{(i}^{\ I}\partial_\mu \Pi_{Ij)} \\
F_{2I}{}^J &= dB_{1I}{}^J + B_{0I}{}^J dA\,, \qquad F_{1I}{}^J = dB_{0I}{}^J \\
G_{\mu\nu\rho I} &= 3\partial_{[\mu}S_{\nu\rho]I} - 12 B_{[\mu I}{}^J G_{\nu\rho]J}
\end{aligned} \tag{3.6}$$

and

$$P^{\mu\nu\rho\sigma\lambda\tau IJ} = \frac{e}{2}e^{-\frac{2\sigma}{\sqrt{10}}}\Pi^{-1}{}_i^{\ I}\Pi^{-1\,iJ} g^{\mu\sigma}g^{\nu\lambda}g^{\rho\tau} + \frac{1}{6}\delta^{IK}\epsilon^{\mu\nu\rho\sigma\lambda\tau} B_{0K}{}^J\,. \tag{3.7}$$

In order to make contact with the maximal D=6 supergravity of Tanii [11] the two form potential S_{2I} must be dualised to another two form potential C_{2I}. The relevant sector of the lagrangean is

$$\mathcal{L} = \frac{1}{144}\left[\epsilon^{\alpha\beta\gamma\delta\epsilon\eta}(G_{\delta\epsilon\eta}{}^I)\right] P^{-1}_{\alpha\beta\gamma\mu\nu\rho IJ}\left[\epsilon^{\mu\nu\rho\sigma\lambda\tau}(G_{\sigma\lambda\tau}{}^J)\right] + \frac{3}{2}\epsilon^{\mu\nu\rho\sigma\lambda\tau}\partial_\mu S_{\nu\rho I} G_{\sigma\lambda}{}^I A_\tau\,. \tag{3.8}$$

To dualise S_{2I} we replace $3\partial_\mu S_{\nu\rho I}$ by the independent field $a_{\mu\nu\rho I}$ and add to the lagrangean the term:

$$\Delta\mathcal{L} = \kappa\epsilon^{\mu\nu\rho\sigma\lambda\tau} a_{\mu\nu\rho I} H_{\sigma\lambda\tau}{}^I\,, \tag{3.9}$$

where κ is a constant and $H_{\sigma\lambda\tau}{}^I = 3\partial_\sigma C_{\lambda\tau}{}^I$. Variation of $\mathcal{L} + \Delta\mathcal{L}$ w.r.t. $a_{\mu\nu\rho I}$ gives

$$\frac{1}{72}P^{-1}_{\sigma\lambda\tau\alpha\beta\gamma IJ}\left[\epsilon^{\alpha\beta\gamma\delta\epsilon\eta}\left(a_{\delta\epsilon\eta}{}^J - 12 B_\delta{}^{JK} G_{\epsilon\eta K}\right)\right] = \kappa H'_{\sigma\lambda\tau I}\,, \tag{3.10}$$

where $H'_{\sigma\lambda\tau I} = H_{\sigma\lambda\tau I} + \frac{1}{2\kappa} G_{\sigma\lambda I} A_\tau$. Substituting back into $\mathcal{L} + \Delta\mathcal{L}$ one finds

$$\mathcal{L} + \Delta\mathcal{L} = -\frac{\kappa^2}{4}(144) H'_{\mu\nu\rho I} P^{\mu\nu\rho\sigma\lambda\tau IJ} H'_{\sigma\lambda\tau J} - 12\kappa \epsilon^{\mu\nu\rho\sigma\lambda\tau} H'_{\mu\nu\rho I} B_\sigma{}^{IJ} G_{\lambda\tau J} \,. \tag{3.11}$$

Thus using the expression for $P^{\mu\nu\rho\sigma\lambda\tau IJ}$ (3.7), $\mathcal{L} + \Delta\mathcal{L}$ becomes

$$\begin{aligned} \mathcal{L} + \Delta\mathcal{L} = &-18\kappa^2 e e^{-\frac{2\sigma}{\sqrt{10}}} \left(\Pi^{-1}{}_i{}^I H'_{\mu\nu\rho I}\right)^2 - 6\kappa^2 \epsilon^{\mu\nu\rho\sigma\lambda\tau} B_0{}^{IJ} H'_{\mu\nu\rho I} H'_{\sigma\lambda\tau J} - \\ &-12\kappa \epsilon^{\mu\nu\rho\sigma\lambda\tau} H'_{\mu\nu\rho I} B_\sigma{}^{IJ} G_{\lambda\tau J} \,. \end{aligned} \tag{3.12}$$

Hence all mention of P^{-1} has disappeared. Therefore finally the bosonic sector of the ungauged D=6 model is (after a simple rescaling of some fields, dropping primes on H'_3 and choosing $\kappa = 1/3$):

$$\begin{aligned} e^{-1}\mathcal{L}_6 = &\, R - \frac{1}{4} e^{-\frac{5\sigma}{\sqrt{10}}} (f_{\mu\nu})^2 - \frac{1}{4} e^{-\frac{\sigma}{\sqrt{10}}} \left(\Pi_I{}^i \Pi_J{}^j F_{\mu\nu}^{IJ}\right)^2 - \frac{1}{4} e^{\frac{3\sigma}{\sqrt{10}}} \left(\Pi^{-1}{}_i{}^I G_{\mu\nu I}\right)^2 - \\ &- \frac{1}{12} e^{-\frac{2\sigma}{\sqrt{10}}} \left(\Pi^{-1}{}_i{}^I H_{\mu\nu\rho I}\right)^2 - P_{\mu ij} P^{\mu ij} - \frac{1}{2} e^{\frac{4\sigma}{\sqrt{10}}} \left(\Pi_I{}^i \Pi_J{}^j F_\mu^{IJ}\right)^2 - \frac{1}{2} (\partial_\mu \sigma)^2 - \\ &- \frac{e^{-1}}{36\sqrt{2}} \epsilon^{\mu\nu\rho\sigma\lambda\tau} B_0{}^{IJ} H_{\mu\nu\rho I} H_{\sigma\lambda\tau J} - \frac{e^{-1}}{6\sqrt{2}} \epsilon^{\mu\nu\rho\sigma\lambda\tau} H_{\mu\nu\rho I} B_\sigma{}^{IJ} G_{\lambda\tau J} \,, \end{aligned} \tag{3.13}$$

where

$$\begin{aligned} f_2 &= dA \qquad G_{2I} = dS_{1I} \\ P_{\mu ij} &= \Pi^{-1}{}_{(i}{}^I \partial_\mu \Pi_{Ij)} \\ F_{2I}{}^J &= dB_{1I}{}^J + B_{0I}{}^J dA \qquad F_{1I}{}^J = dB_{0I}{}^J \\ H_{\mu\nu\rho I} &= 3\left(\partial_{[\mu} C_{\nu\rho]I} + \frac{1}{2} G_{[\mu\nu I} A_{\rho]}\right) . \end{aligned} \tag{3.14}$$

The bosonic field content of this model is 1 graviton, (14+10+1) scalars, (10+5+1) vectors and 5 second rank antisymmetric tensors. This is precisely the bosonic field content of the maximally extended supergravity in six dimensions [11]. Comparison of the interactions is complicated by the manifestly SO(5,5) invariant formulation of the ungauged model [11] but the interaction terms in (3.13) seem to have at least qualitatively the correct structure. As all local supersymmetries of the action remain unbroken after S^1 reduction, the lagrangean (3.13) must have a supersymmetric extension. In appendix C we present the complete lagrangean and supersymmetry transformation laws of this D=6 maximal supergravity.

As this section has been concerned with limits in which gauge coupling constants in gauged supergravities are turned off, we take this opportunity to note that contrary to the statement in [16], the limit in which the U(1) coupling constant of D=5 N=4 SU(2)×U(1) gauged supergravity is turned off, can be recovered. This results in the D=5 N=4 SU(2) gauged model obtained by the reduction on T^2 of D=7 N=2 SU(2) gauged supergravity [17, 18]. In appendix D we verify this statement.

4. Conclusion

We have reduced to six dimensions the maximal D=7 SO(5) gauged supergravity [8] giving details of the reduction of the bosonic sector only. After eliminating some auxiliary fields one obtains a new SO(5) gauged supergravity whose field content is 1 graviton, 5 massive 2-index tensor potentials, 11 vectors, 25 scalars, 4 gravitini and 20 spin 1/2 fermions. As all local supersymmetries of the action remain unbroken after reduction on S^1 this reduced D=6 model must have a supersymmetric extension which we have presented in an appendix. Since the D=7 SO(5) gauged supergravity can be obtained from eleven dimensional supergravity by compactification on S^4, this new D=6 SO(5) gauged supergravity is then the compactification on $S^4 \times S^1$ of D=11 supergravity. Performing this reduction with the S^1 and S^4 factors in reverse order must lead to the same D=6 SO(5) gauged model. Therefore the reduction of D=10 N=2a supergravity on S^4 must also yield the D=6 SO(5) gauged model presented in this paper, and it would be interesting to verify this. The existence of an S^4 compactification of D=10 IIa supergravity was also concluded in [2] through the discovery of a dual frame in which the near horizon geometry of the D4-brane supergravity solution is $AdS_6 \times S^4$. Having obtained a new D=6 SO(5) gauged supergravity, we note that one may also perform further S^1 reductions to yield new gauged maximal supergravities in D=5 and D=4.

It is interesting that, unlike its seven dimensional progenitor, our new D=6 SO(5) gauged supergravity has no 'gauge discontinuity'. We have shown that after regurgitating five vectors from the 2-index tensor potentials, a procedure which is essentially the reverse of the Higgs mechanism, one can take the $g \to 0$ limit in the bosonic sector. The bosonic field rescalings (3.2) necessary to show this are also sufficient to ensure the $g = 0$ limit is obtainable without problems in the fermionic sector and supersymmetry transformation laws.

The resulting model has a field content of 1 graviton, 5 massless 2-index tensor potentials, 16 vectors, 25 scalars, 4 gravitini and 20 spin 1/2 fermions and is presumably equivalent to the maximally extended supergravity of Tanii [11] in which all the internal symmetries are manifest.

As shown in [9] there exist non-compact maximal gauged supergravities in D=7 with gauge groups SO(4,1) and SO(3,2). These models are simply obtained from the SO(5) model by replacing all gauge fields by SO(p,q) gauge fields and all δ^{IJ} contractions with η^{IJ} contractions where η^{IJ}=diag$(----+)$ or diag$(+++--)$ for SO(4,1) or SO(3,2) respectively. Hence these models also have gauge discontinuities and the results for the dimensionally reduced SO(5) gauged model are easily extended to these non-compactly gauged models. The dimensionally reduced SO(4,1) model may also possess a limit in D=6 where the gauging is only partially switched off and SO(4,1) is contracted to ISO(4).

As mentioned in the introduction, the scalar potential of the D=7 SO(5) gauged supergravity has a supersymmetry preserving maximum which is AdS_7 [9]. It was pointed out in [13] that Anti-De Sitter space upon S^1 dimensional reduction gives a domain wall preserving 1/2 of the supersymmetry. Therefore one expects the D=6 SO(5) gauged model (2.15) to possess a 1/2 supersymmetric domain wall vacuum. We have shown this to be the case by solving the D=6 Killing spinor equations. Reduction of the D=7 supersymmetry transformation law of the gravitino using the ansatz (2.4) with $A_\mu = 0$ yields the D=6 Killing spinor equations:

$$\begin{aligned} \delta\chi &= \frac{1}{2\sqrt{2}}\tau^\mu\tau^{\underline{7}}\epsilon\partial_\mu\sigma - \frac{5m}{4\sqrt{10}}\tau^{\underline{7}}\epsilon e^{\frac{\sigma}{2\sqrt{10}}} \\ \delta\psi_\mu &= \partial_\mu\epsilon + \frac{1}{4}\omega_{\mu\underline{\nu}\underline{\rho}}\tau^{\underline{\nu}\underline{\rho}}\epsilon - \frac{5\sqrt{2}m}{16}\tau_\mu\epsilon e^{\frac{\sigma}{2\sqrt{10}}} + \frac{1}{2\sqrt{10}}\partial_\mu\sigma\epsilon\,, \end{aligned} \tag{4.1}$$

where $\mu = 0, \ldots, 5$, underlined indices refer to 'flat' space and $\tau^{\underline{7}}$ is the product of all of the D=6 Dirac matrices. Provided $\epsilon = H^{\frac{1}{16}}(y)\epsilon_0$ where $\tau_{\underline{y}}\epsilon_0 = \pm\epsilon_0$, these equations are satisfied in the domain wall background

$$\begin{aligned} ds^2 &= H^{-\frac{5}{12}}dx^\alpha dx^\beta\eta_{\alpha\beta} + H^{-\frac{25}{12}}dy^2 \\ e^\sigma &= H^{\frac{\sqrt{10}}{12}}\,, \end{aligned} \tag{4.2}$$

where $H = c + 3\sqrt{2}m|y|$ with c a positive constant and $\alpha, \beta = 0, \ldots, 4$. Hence this domain wall preserves 1/2 of the supersymmetry. See refs [13, 19, 20, 21] for more details of domain wall solutions in higher dimensional supergravities.

Further gauged maximal supergravities can also be found from dimensional reduction of those gauged supergravities in D=7 and D=8. The truncation of these models to non-maximal gauged supergravities is also interesting. For example, the S^1 reduction and truncation of D=8 SU(2) gauged supergravity [10] may yield the D=7 SU(2) gauged simple supergravity [18]. More interestingly, the Scherk-Schwarz reduction of the D=8 model may provide a way of obtaining the version of D=7 SU(2) gauged supergravity which has a topological mass term. In addition to reductions on tori, one may obtain new gauged maximal supergravities from reductions on spheres of those models in D=7 and D=8. For example, it has recently been proposed [22] that the further reduction of D=8 SU(2) gauged supergravity on S^3 would yield a D=5 SU(2)×SU(2) maximal supergravity. That these reductions, leading to new gauged supergravities, are possible, can be attributed to the existence of supersymmetric intersecting brane solutions in D=11 and D=10 having near horizon geometries of the form $\mathrm{AdS}_k\times\mathrm{S}^l\times\mathrm{S}^m \times \mathbb{E}^n$.

In appendix D we have also shown that the limit of the bosonic sector of D=5 N=4 SU(2)×U(1) gauged supergravity, in which the U(1) coupling constant is turned off, is the D=5 N=4 SU(2) gauged model obtained in [17].

Acknowledgments

The author takes enormous pleasure in thanking Paul Townsend for constant encouragement and criticisms.

A. $D = 6$ SO(5) gauged supergravity

Here we present the complete lagrangean and supersymmetry transformation laws of D=6 SO(5) gauged supergravity. The field content is 1 graviton $g_{\mu\nu}$, 5 2-index massive antisymmetric tensor potentials $S_{\mu\nu I}$, (10+1) vectors $({B_{\mu I}}^J, A_\mu)$, (14+10+1) scalars $({\Pi_I}^j, {B_{0I}}^J, \sigma)$, 4 gravitini ψ_μ and (16+4) spin 1/2 fermions (λ_i, χ). ψ_μ, λ_i and χ are all D=6 USp(4) symplectic Majorana spinors and the spinor inversion formula reads

$$\bar{\lambda}\tau^{\alpha_1\ldots\alpha_n}\gamma^{i_1\ldots i_r}\chi = (-1)^{\frac{n(n+1)}{2}}(-1)^{\frac{r(r-1)}{2}}\bar{\chi}\tau^{\alpha_1\ldots\alpha_n}\gamma^{i_1\ldots i_r}\lambda\,, \tag{A.1}$$

where γ^i are the D=5 Dirac matrices. The lagrangean of D=6 SO(5) supergravity, neglecting quartic fermion terms, is:

$$\begin{aligned}
e^{-1}\mathcal{L}_6 = & R - \frac{1}{4}e^{-\frac{5\sigma}{\sqrt{10}}}(f_{\mu\nu})^2 - \frac{1}{4}e^{-\frac{\sigma}{\sqrt{10}}}\left({\Pi_I}^i{\Pi_J}^j F^{IJ}_{\mu\nu}\right)^2 - P_{\mu ij}P^{\mu ij} - \frac{1}{2}e^{\frac{4\sigma}{\sqrt{10}}}\left({\Pi_I}^i{\Pi_J}^j F^{IJ}_{\mu}\right)^2 - \\
& - \frac{1}{2}(\partial_\mu\sigma)^2 - m^2 e^{\frac{5\sigma}{\sqrt{10}}}Q_{(ij)}Q^{(ij)} + m^2 e^{\frac{\sigma}{\sqrt{10}}}(T^2 - 2T_{ij}T^{ij}) - 3m^2 e^{\frac{3\sigma}{\sqrt{10}}}\left({\Pi^{-1}_i}^I S_{\mu\nu I}\right)^2 + \\
& + \frac{3e^{-1}m}{4\sqrt{2}}\epsilon^{\mu\nu\rho\sigma\lambda\tau}{S_{\mu\nu}}^I S_{\rho\sigma I}\partial_\lambda A_\tau - \frac{\sqrt{3}ie^{-1}}{2\sqrt{2}}\epsilon_{JKLMN}\epsilon^{\mu\nu\rho\sigma\lambda\tau}{S_{\mu\nu}}^J F^{KL}_{\rho\sigma}F^{MN}_{\lambda\tau} + \\
& + \frac{e^{-1}}{144}\left[\epsilon^{\alpha\beta\gamma\delta\epsilon\eta}{\tilde{G}_{\delta\epsilon\eta}}^I\right]P^{-1}_{\alpha\beta\gamma\mu\nu\rho IJ}\left[\epsilon^{\mu\nu\rho\sigma\lambda\tau}{\tilde{G}_{\sigma\lambda\tau}}^J\right] + \frac{e^{-1}}{2\sqrt{2}m}\Omega(B) - \\
& - \bar{\psi}_\mu\tau^{\mu\nu\rho}\nabla_\nu\psi_\rho - \bar{\chi}\tau^\mu\nabla_\mu\chi - \bar{\lambda}^i\tau^\mu\nabla_\mu\lambda_i + \frac{m}{2\sqrt{2}}T\bar{\psi}_\mu\tau^{\mu\nu}\psi_\nu e^{\frac{\sigma}{2\sqrt{10}}} - \\
& - \frac{m}{2\sqrt{10}}T\bar{\psi}_\mu\tau^\mu\tau^{\underline{7}}\chi e^{\frac{\sigma}{2\sqrt{10}}} - \frac{9m}{20\sqrt{2}}T\bar{\chi}\chi e^{\frac{\sigma}{2\sqrt{10}}} - \frac{m}{2\sqrt{2}}\left[8T^{ij} - T\delta^{ij}\right]\bar{\lambda}_i\lambda_j e^{\frac{\sigma}{2\sqrt{10}}} + \\
& + \sqrt{2}mT^{ij}\bar{\lambda}_i\gamma_j\tau^\mu\psi_\mu e^{\frac{\sigma}{2\sqrt{10}}} - \frac{2m}{\sqrt{10}}T^{ij}\bar{\lambda}_i\gamma_j\tau^{\underline{7}}\chi e^{\frac{\sigma}{2\sqrt{10}}} + \bar{\psi}_\mu\tau^\nu\tau^\mu\gamma^i\lambda^j P_{\nu ij} + \\
& + 2m\bar{\psi}_\mu\tau^{\underline{7}}\tau^\mu\gamma^i\lambda^j Q_{(ij)}e^{\frac{5\sigma}{2\sqrt{10}}} + 2\sqrt{5}m\bar{\chi}\gamma^i\lambda^j Q_{(ij)}e^{\frac{5\sigma}{2\sqrt{10}}} + \frac{1}{4\sqrt{10}}\bar{\psi}_\mu g^{\mu\nu}\tau^\lambda\psi_\nu\partial_\lambda\sigma - \\
& - \frac{1}{\sqrt{2}}\bar{\psi}_\mu\tau^\nu\tau^\mu\tau^{\underline{7}}\chi\partial_\nu\sigma - \frac{1}{8}\bar{\psi}_\mu\left[2\tau^{\mu\nu\rho}\tau^\lambda + \tau^{\mu\rho}\tau^{\nu\lambda}\right]\tau^{\underline{7}}\psi_\rho f_{\nu\lambda}e^{-\frac{5\sigma}{2\sqrt{10}}} - \\
& - \frac{\sqrt{5}}{8}\bar{\psi}_\mu\tau^{\nu\lambda}\tau^\mu\chi f_{\nu\lambda}e^{-\frac{5\sigma}{2\sqrt{10}}} - \frac{3}{16}\bar{\chi}\tau^{\underline{7}}\tau^{\nu\lambda}\chi f_{\nu\lambda}e^{-\frac{5\sigma}{2\sqrt{10}}} - \frac{1}{8}\bar{\lambda}^i\tau^{\underline{7}}\tau^{\nu\lambda}\lambda_i f_{\nu\lambda}e^{-\frac{5\sigma}{2\sqrt{10}}} + \\
& + \frac{1}{8\sqrt{2}}\bar{\psi}_\mu\left[\tau^{\mu\nu\rho\sigma} - 2g^{\mu\nu}g^{\rho\sigma}\right]\gamma_{ij}\psi_\sigma{\Pi_I}^i{\Pi_J}^j F^{IJ}_{\nu\rho}e^{-\frac{\sigma}{2\sqrt{10}}} + \\
& + \frac{1}{8\sqrt{10}}\bar{\psi}_\mu\left[2g^{\mu\nu}\tau^\rho + \tau^{\mu\nu\rho}\right]\tau^{\underline{7}}\gamma_{ij}\chi{\Pi_I}^i{\Pi_J}^j F^{IJ}_{\nu\rho}e^{-\frac{\sigma}{2\sqrt{10}}} -
\end{aligned}$$

$$
\begin{aligned}
&-\frac{11}{80\sqrt{2}}\bar\chi\tau^{\nu\rho}\gamma_{ij}\chi\Pi_I{}^i\Pi_J{}^jF^{IJ}_{\nu\rho}e^{-\frac{\sigma}{2\sqrt{10}}} + \frac{1}{2\sqrt{2}}\bar\psi_\mu\tau^{\nu\rho}\tau^\mu\gamma_i\lambda_j\Pi_I{}^i\Pi_J{}^jF^{IJ}_{\nu\rho}e^{-\frac{\sigma}{2\sqrt{10}}} -\\
&-\frac{1}{2\sqrt{10}}\bar\chi\tau^{\nu\rho}\tau^{\underline{7}}\gamma_i\lambda_j\Pi_I{}^i\Pi_J{}^jF^{IJ}_{\nu\rho}e^{-\frac{\sigma}{2\sqrt{10}}}+\frac{1}{16\sqrt{2}}\bar\lambda_i\gamma^j\gamma_{kl}\gamma^i\tau^{\nu\rho}\lambda_j\Pi_I{}^k\Pi_J{}^lF^{IJ}_{\nu\rho}e^{-\frac{\sigma}{2\sqrt{10}}} -\\
&-\frac{1}{4\sqrt{2}}\bar\psi_\mu\tau^{\mu\nu\rho}\tau^{\underline{7}}\gamma_{ij}\psi_\rho\Pi_I{}^i\Pi_J{}^jF_\nu^{IJ}e^{\frac{2\sigma}{\sqrt{10}}} - \frac{1}{\sqrt{10}}\bar\psi_\mu\tau^\mu\tau^\nu\gamma_{ij}\chi\Pi_I{}^i\Pi_J{}^jF_\nu^{IJ}e^{\frac{2\sigma}{\sqrt{10}}} +\\
&+\frac{7}{20\sqrt{2}}\bar\chi\tau^{\underline{7}}\tau^\nu\gamma_{ij}\chi\Pi_I{}^i\Pi_J{}^jF_\nu^{IJ}e^{\frac{2\sigma}{\sqrt{10}}} - \frac{1}{\sqrt{2}}\bar\psi_\mu\tau^\nu\tau^\mu\tau^{\underline{7}}\gamma_i\lambda_j\Pi_I{}^i\Pi_J{}^jF_\nu^{IJ}e^{\frac{2\sigma}{\sqrt{10}}} +\\
&+\frac{4}{\sqrt{10}}\bar\chi\tau^\nu\gamma_i\lambda_j\Pi_I{}^i\Pi_J{}^jF_\nu^{IJ}e^{\frac{2\sigma}{\sqrt{10}}} + \frac{1}{8\sqrt{2}}\bar\lambda_i\gamma^j\gamma_{kl}\gamma^i\tau^\nu\tau^{\underline{7}}\lambda_j\Pi_I{}^k\Pi_J{}^lF_\nu^{IJ}e^{\frac{2\sigma}{\sqrt{10}}} -\\
&-\frac{im\sqrt{3}}{4}\bar\psi_\mu\left[\tau^{\mu\nu\rho\lambda}+g^{\mu\nu}g^{\rho\lambda}\right]\tau^{\underline{7}}\gamma^i\psi_\lambda\Pi^{-1}{}_i{}^IS_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}} -\\
&-\frac{3im\sqrt{3}}{4\sqrt{5}}\bar\psi_\mu\left[\tau^{\mu\nu\rho}-2g^{\mu\nu}\tau^\rho\right]\gamma^i\chi\Pi^{-1}{}_i{}^IS_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}} -\\
&-\frac{i\sqrt{3}}{40}\bar\chi\tau^{\underline{7}}\tau^{\nu\rho}\gamma^i\chi\Pi^{-1}{}_i{}^IS_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}} - \frac{im\sqrt{3}}{2}\bar\psi_\mu\left[\tau^{\mu\nu\rho}-2g^{\mu\nu}\tau^\rho\right]\tau^{\underline{7}}\lambda^i\Pi^{-1}{}_i{}^IS_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}}+\\
&+\frac{9im}{2\sqrt{15}}\bar\chi\tau^{\nu\rho}\lambda^i\Pi^{-1}{}_i{}^IS_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}} - \frac{im\sqrt{3}}{4}\bar\lambda^j\tau^{\nu\rho}\tau^{\underline{7}}\gamma^i\lambda_j\Pi^{-1}{}_i{}^IS_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}} +\\
&+\frac{m}{2}\bar\psi_\mu\tau^{\mu\nu}\tau^{\underline{7}}Q_{[ij]}\gamma^{ij}\psi_\nu e^{\frac{5\sigma}{2\sqrt{10}}}+\frac{m\sqrt{5}}{2}\bar\psi_\mu\tau^\mu Q_{[ij]}\gamma^{ij}\chi e^{\frac{5\sigma}{2\sqrt{10}}} - \frac{3m}{4}\bar\chi\tau^{\underline{7}}Q_{[ij]}\gamma^{ij}\chi e^{\frac{5\sigma}{2\sqrt{10}}} -\\
&-\frac{m}{2}\bar\lambda^k\tau^{\underline{7}}Q_{[ij]}\gamma^{ij}\lambda_k e^{\frac{5\sigma}{2\sqrt{10}}} - 2m\bar\lambda^i\tau^{\underline{7}}Q_{[ij]}\lambda^j e^{\frac{5\sigma}{2\sqrt{10}}} +\\
&+\frac{i}{24\sqrt{6}}\Bigg\{\frac{1}{2}\bar\psi_\sigma\left[\tau^{\sigma\mu\nu\rho\lambda}+6g^{\sigma\mu}\tau^\nu g^{\rho\lambda}\right]\gamma^i\psi_\lambda+\frac{1}{\sqrt{5}}\bar\psi_\sigma\left[\tau^{\sigma\mu\nu\rho}-3g^{\sigma\mu}\tau^{\nu\rho}\right]\tau^{\underline{7}}\gamma^i\chi +\\
&\qquad+\frac{3}{10}\bar\chi\tau^{\mu\nu\rho}\gamma^i\chi - \bar\psi_\sigma\left[\tau^{\sigma\mu\nu\rho}-3g^{\sigma\mu}\tau^{\nu\rho}\right]\lambda^i - \frac{2}{\sqrt{5}}\bar\chi\tau^{\underline{7}}\tau^{\mu\nu\rho}\lambda^i -\\
&\qquad-\frac{1}{2}\bar\lambda^j\tau^{\mu\nu\rho}\gamma^i\lambda_j\Bigg\}\Pi^{-1}{}_i{}^IP^{-1}_{\mu\nu\rho\delta\epsilon\eta IJ}\left[\epsilon^{\delta\epsilon\eta\alpha\beta\gamma}\tilde G_{\alpha\beta\gamma}{}^J\right]e^{-\frac{\sigma}{\sqrt{10}}}\,, \qquad \text{(A.2)}
\end{aligned}
$$

where

$$
\begin{aligned}
T_{ij} &= \Pi^{-1}{}_i{}^I\Pi^{-1}{}_j{}^J\delta_{IJ} \qquad T = T_{ij}\delta^{ij} \qquad f_2 = dA\\
F_{2I}{}^J &= dB_{1I}{}^J + B_{0I}{}^J dA + gB_{1I}{}^KB_{1K}{}^J\\
F_{1I}{}^J &= dB_{0I}{}^J + g\left(B_{1I}{}^KB_{0K}{}^J - B_{0I}{}^KB_{1K}{}^J\right)\\
G_{3I} &= dS_{2I} + gB_{1I}{}^JS_{2J}\\
\tilde G_{\mu\nu\rho I} &= G_{\mu\nu\rho I} - \frac{\sqrt{3}i}{2g}\epsilon_{IKLMN}F^{KL}_{\mu\nu}F^{MN}_\rho\\
P_{\mu ij} &= \Pi^{-1}{}_{(i}{}^I\left(\delta_I{}^J\partial_\mu + gB_{\mu I}{}^J\right)\Pi_J{}^k\delta_{j)k}\\
Q_{\mu ij} &= \Pi^{-1}{}_{[i}{}^I\left(\delta_I{}^J\partial_\mu + gB_{\mu I}{}^J\right)\Pi_J{}^k\delta_{j]k}\\
Q_{ij} &= \Pi^{-1}{}_i{}^IB_{0I}{}^J\Pi_J{}^k\delta_{jk}\\
P^{\mu\nu\rho\sigma\lambda\tau IJ} &= \frac{e}{2}e^{-\frac{2\sigma}{\sqrt{10}}}\Pi^{-1}{}_i{}^I\Pi^{-1\,iJ}g^{\mu\sigma}g^{\nu\lambda}g^{\rho\tau} + \frac{1}{6\sqrt{2}}\delta^{IK}\epsilon^{\mu\nu\rho\sigma\lambda\tau}B_{0K}{}^J\,, \qquad \text{(A.3)}
\end{aligned}
$$

and

$$\nabla_\mu \xi_{\nu i} = \partial_\mu \xi_{\nu i} + \frac{1}{4}\omega_{\mu\underline{\nu\rho}}\tau^{\underline{\nu\rho}}\xi_{\nu i} + \Gamma^\rho_{\mu\nu}\xi_{\rho i} + \frac{1}{4}Q_{\mu jk}\gamma^{jk}\xi_{\nu i} + Q_{\mu i}{}^{j}\xi_{\nu j}\,, \tag{A.4}$$

for a general vector spinor, $\xi_{\nu i}$, of both SO(5,1) and SO(5)$_c$. ψ_μ and χ are SO(5)$_c$ spinors and λ_i is an SO(5)$_c$ vector spinor. The supersymmetry transformations of the fields neglecting terms cubic in fermion fields are:

$$\begin{aligned}
\delta\psi_\mu = {} & \nabla_\mu\epsilon - \frac{\sqrt{2}m}{16}T\tau_\mu\epsilon e^{\frac{\sigma}{2\sqrt{10}}} + \frac{m}{8}\tau_\mu\tau^{\underline{7}}Q_{[ij]}\gamma^{ij}\epsilon e^{\frac{5\sigma}{2\sqrt{10}}} - \\
& -\frac{1}{32}\left[\tau_\mu{}^{\nu\rho} - 6\delta^\nu_\mu\tau^\rho\right]\tau^{\underline{7}}\epsilon f_{\nu\rho}e^{-\frac{5\sigma}{2\sqrt{10}}} - \\
& -\frac{1}{32\sqrt{2}}\left[\tau_\mu{}^{\nu\rho} - 6\delta^\nu_\mu\tau^\rho\right]\gamma_{ij}\epsilon\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\nu\rho}e^{-\frac{\sigma}{2\sqrt{10}}} + \frac{1}{4\sqrt{2}}\tau^{\underline{7}}\gamma_{ij}\epsilon\Pi_I{}^i\Pi_J{}^j F^{IJ}_\mu e^{\frac{2\sigma}{\sqrt{10}}} + \\
& +\frac{1}{2\sqrt{10}}\epsilon\partial_\mu\sigma + \frac{im\sqrt{3}}{16}\left[\tau_\mu{}^{\nu\rho} - 6\delta^\nu_\mu\tau^\rho\right]\tau^{\underline{7}}\gamma^i\epsilon\Pi^{-1}{}_i{}^I S_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}} + \\
& +\frac{i}{96\sqrt{6}}\left[\tau_\mu{}^{\nu\rho\sigma} - 3\delta^\nu_\mu\tau^{\rho\sigma}\right]\gamma^i\epsilon\Pi^{-1}{}_i{}^I P^{-1}_{\nu\rho\sigma\alpha\beta\gamma IJ}\left[\epsilon^{\alpha\beta\gamma\delta\epsilon\eta}\tilde{G}_{\delta\epsilon\eta}{}^J\right]e^{-\frac{\sigma}{\sqrt{10}}}\,,
\end{aligned} \tag{A.5}$$

$$\begin{aligned}
\delta\chi = {} & -\frac{\sqrt{5}}{16}\tau^{\mu\nu}\epsilon f_{\mu\nu}e^{-\frac{5\sigma}{2\sqrt{10}}} + \frac{1}{2\sqrt{2}}\tau^\mu\tau^{\underline{7}}\epsilon\partial_\mu\sigma - \frac{m}{4\sqrt{10}}T\tau^{\underline{7}}\epsilon e^{\frac{\sigma}{2\sqrt{10}}} - \\
& -\frac{1}{16\sqrt{10}}\tau^{\mu\nu}\tau^{\underline{7}}\gamma_{ij}\epsilon\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\mu\nu}e^{-\frac{\sigma}{2\sqrt{10}}} - \frac{1}{2\sqrt{10}}\tau^\mu\gamma_{ij}\epsilon\Pi_I{}^i\Pi_J{}^j F^{IJ}_\mu e^{\frac{2\sigma}{\sqrt{10}}} - \\
& -\frac{9im}{8\sqrt{15}}\tau^{\mu\nu}\gamma^i\epsilon\Pi^{-1}{}_i{}^I S_{\mu\nu I}e^{\frac{3\sigma}{2\sqrt{10}}} + \frac{m\sqrt{5}}{4}Q_{[ij]}\gamma^{ij}\epsilon e^{\frac{5\sigma}{2\sqrt{10}}} - \\
& -\frac{i}{48\sqrt{30}}\tau^{\mu\nu\rho}\tau^{\underline{7}}\gamma^i\epsilon\Pi^{-1}{}_i{}^I P^{-1}_{\mu\nu\rho\alpha\beta\gamma IJ}\left[\epsilon^{\alpha\beta\gamma\delta\epsilon\eta}\tilde{G}_{\delta\epsilon\eta}{}^J\right]e^{-\frac{\sigma}{\sqrt{10}}}\,,
\end{aligned} \tag{A.6}$$

$$\begin{aligned}
\delta\lambda_i = {} & \frac{1}{16\sqrt{2}}\tau^{\mu\nu}\left[\gamma_{kl}\gamma_i - \frac{1}{5}\gamma_i\gamma_{kl}\right]\epsilon\Pi_I{}^k\Pi_J{}^l F^{IJ}_{\mu\nu}e^{-\frac{\sigma}{2\sqrt{10}}} + \frac{1}{2}\tau^\mu\gamma^j\epsilon P_{\mu ij} + \\
& +\frac{1}{8\sqrt{2}}\tau^\mu\tau^{\underline{7}}\left[\gamma_{kl}\gamma_i - \frac{1}{5}\gamma_i\gamma_{kl}\right]\epsilon\Pi_I{}^k\Pi_J{}^l F^{IJ}_\mu e^{\frac{2\sigma}{\sqrt{10}}} + m\tau^{\underline{7}}\gamma^j\epsilon Q_{(ij)}e^{\frac{5\sigma}{2\sqrt{10}}} + \\
& +\frac{3im}{20\sqrt{3}}\tau^{\mu\nu}\tau^{\underline{7}}\left[\gamma_i{}^j - 4\delta_i^j\right]\epsilon\Pi^{-1}{}_j{}^I S_{\mu\nu I}e^{\frac{3\sigma}{2\sqrt{10}}} + \frac{m}{\sqrt{2}}\left[T_{ij} - \frac{1}{5}\delta_{ij}T\right]\gamma^j\epsilon e^{\frac{\sigma}{2\sqrt{10}}} + \\
& +\frac{i}{240\sqrt{6}}\tau^{\mu\nu\rho}\left[\gamma_i{}^j - 4\delta_i^j\right]\epsilon\Pi^{-1}{}_j{}^I P^{-1}_{\mu\nu\rho\alpha\beta\gamma IJ}\left[\epsilon^{\alpha\beta\gamma\delta\epsilon\eta}\tilde{G}_{\delta\epsilon\eta}{}^J\right]e^{-\frac{\sigma}{\sqrt{10}}}\,,
\end{aligned} \tag{A.7}$$

$$\delta\sigma = -\frac{1}{\sqrt{2}}\bar{\epsilon}\tau^{\underline{7}}\chi e^{\frac{\sigma}{\sqrt{10}}}\,, \qquad \Pi^{-1}{}_i{}^I\delta\Pi_I{}^j = \frac{1}{4}\left[\bar{\epsilon}\gamma_i\lambda^j + \bar{\epsilon}\gamma^j\lambda_i\right]e^{\frac{\sigma}{\sqrt{10}}}, \tag{A.8}$$

$$\delta A_\mu = \frac{1}{2}\bar{\epsilon}\tau^{\underline{7}}\psi_\mu e^{\frac{7\sigma}{2\sqrt{10}}} + \frac{1}{4\sqrt{5}}\bar{\epsilon}\tau_\mu\chi e^{\frac{7\sigma}{2\sqrt{10}}}\,, \tag{A.9}$$

$$\delta e^{\underline{\mu}}{}_\mu = \frac{1}{2}\bar{\epsilon}\tau^{\underline{\mu}}\psi_\mu e^{\frac{\sigma}{\sqrt{10}}} + \frac{1}{\sqrt{5}}\bar{\epsilon}\tau^{\underline{\mu}}\chi A_\mu e^{-\frac{3\sigma}{2\sqrt{10}}} + \frac{1}{4\sqrt{5}}\bar{\epsilon}\tau_\mu{}^\nu e^{\underline{\mu}}{}_\nu\tau^{\underline{7}}\chi e^{\frac{\sigma}{\sqrt{10}}}\,, \tag{A.10}$$

$$
\begin{aligned}
\Pi_I{}^i \Pi_J{}^j \delta B_0{}^{IJ} = {} & \frac{1}{\sqrt{10}} \bar\epsilon \gamma^{ij} \chi e^{-\frac{\sigma}{\sqrt{10}}} - \frac{1}{\sqrt{5}} \bar\epsilon \tau^\mu \chi \Pi_I{}^i \Pi_J{}^j B_\mu{}^{IJ} e^{-\frac{3\sigma}{2\sqrt{10}}} + \\
& + \frac{1}{4\sqrt{2}} \bar\epsilon \tau^{\underline{7}} \gamma^k \gamma^{ij} \lambda_k e^{-\frac{\sigma}{\sqrt{10}}} ,
\end{aligned}
\tag{A.11}
$$

$$
\begin{aligned}
\Pi_I{}^i \Pi_J{}^j \delta B_\mu{}^{IJ} = {} & \frac{1}{2\sqrt{2}} \bar\epsilon \gamma^{ij} \psi_\mu e^{\frac{3\sigma}{2\sqrt{10}}} + \frac{1}{4\sqrt{10}} \bar\epsilon \gamma^{ij} \tau^{\underline{7}} \tau_\mu \chi e^{\frac{3\sigma}{2\sqrt{10}}} + \frac{1}{4\sqrt{2}} \bar\epsilon \tau_\mu \gamma^k \gamma^{ij} \lambda_k e^{\frac{3\sigma}{2\sqrt{10}}} - \\
& - \frac{1}{2} \bar\epsilon \tau^{\underline{7}} \psi_\mu \Pi_I{}^i \Pi_J{}^j B_0{}^{IJ} e^{\frac{7\sigma}{2\sqrt{10}}} - \frac{1}{4\sqrt{5}} \bar\epsilon \tau_\mu \chi \Pi_I{}^i \Pi_J{}^j B_0{}^{IJ} e^{\frac{7\sigma}{2\sqrt{10}}} + \\
& + \frac{1}{\sqrt{5}} \bar\epsilon \tau^\nu A_\mu \chi \Pi_I{}^i \Pi_J{}^j B_\nu{}^{IJ} e^{-\frac{3\sigma}{2\sqrt{10}}} ,
\end{aligned}
\tag{A.12}
$$

$$
\begin{aligned}
\delta S_{\mu\nu I} = {} & -\frac{2}{\sqrt{5}} \bar\epsilon \tau^\rho \chi A_\mu S_{\nu\rho I} e^{-\frac{3\sigma}{2\sqrt{10}}} + \\
& + \frac{i}{2\sqrt{6}} \delta_{IJ} \Pi^{-1}{}_i{}^J \left[\frac{1}{\sqrt{5}} \bar\epsilon \tau_{\mu\nu} \gamma^i \chi - 2 \bar\epsilon \tau_\mu \tau^{\underline{7}} \gamma^i \psi_\nu - \bar\epsilon \tau_{\mu\nu} \tau^{\underline{7}} \lambda^i \right] - \\
& - \frac{i}{8\sqrt{6}m} \Pi_I{}^i \left[\frac{4}{\sqrt{5}} \bar\epsilon \gamma_{ijk} \chi + \bar\epsilon \tau^{\underline{7}} \gamma^l \gamma_{ijk} \lambda_l \right] \Pi_J{}^j \Pi_K^k F_{\mu\nu}^{JK} e^{-\frac{\sigma}{\sqrt{10}}} - \\
& - \frac{i}{8\sqrt{6}m} \Pi_I{}^i \left[4 \bar\epsilon \gamma_{ijk} \psi_\mu + \frac{2}{\sqrt{5}} \bar\epsilon \gamma_{ijk} \tau^{\underline{7}} \tau_\mu \chi + 2 \bar\epsilon \tau_\mu \gamma^l \gamma_{ijk} \lambda_l \right] \Pi_J{}^j \Pi_K^k F_\nu^{JK} e^{\frac{3\sigma}{2\sqrt{10}}} - \\
& - \frac{i\sqrt{3}}{12m} \delta_{IJ} \Pi^{-1}{}_i{}^J \nabla_\mu \left[\frac{3}{\sqrt{5}} \bar\epsilon \tau_\nu \gamma^i \chi e^{-\frac{\sigma}{2\sqrt{10}}} + 2 \bar\epsilon \tau_\nu \tau^{\underline{7}} \lambda^i e^{-\frac{\sigma}{2\sqrt{10}}} - 2 \bar\epsilon \tau^{\underline{7}} \gamma^i \psi_\nu e^{-\frac{\sigma}{2\sqrt{10}}} \right] - \\
& - \frac{i\sqrt{3}}{6} \delta_{IJ} \Pi^{-1}{}_i{}^J Q^{[ij]} \left[2 \bar\epsilon \tau_\mu \gamma_j \psi_\nu - \frac{1}{\sqrt{5}} \bar\epsilon \tau_{\mu\nu} \tau^{\underline{7}} \gamma_j \chi + \bar\epsilon \tau_{\mu\nu} \lambda_j \right] e^{\frac{2\sigma}{\sqrt{10}}} - \\
& - \frac{1}{12\sqrt{10}m} \bar\epsilon \tau^\rho \chi P^{-1}_{\mu\nu\rho\alpha\beta\gamma IJ} \left[\epsilon^{\alpha\beta\gamma\delta\epsilon\eta} \tilde{G}_{\delta\epsilon\eta}{}^J \right] e^{-\frac{3\sigma}{2\sqrt{10}}} .
\end{aligned}
\tag{A.13}
$$

B. The $D = 6$ Chern-Simons term $\Omega(B)$

In this section we present the explicit expression for the D=6 Chern-Simons term $\Omega(B)$[4]

$$
\begin{aligned}
4\Omega(B) = \epsilon^{\mu\nu\rho\sigma\lambda\tau} \mathrm{Tr} \Big\{ & B_\mu F_{\nu\rho} F_{\sigma\lambda} F_\tau + B_\mu F_{\nu\rho} F_\sigma F_{\lambda\tau} + B_\mu F_\nu F_{\rho\sigma} F_{\lambda\tau} + \frac{1}{2} B_0 F_{\mu\nu} F_{\rho\sigma} F_{\lambda\tau} + \\
& - \frac{4}{5} g \Big[B_\mu B_\nu B_\rho F_{\sigma\lambda} F_\tau + B_\mu B_\nu B_\rho F_\sigma F_{\lambda\tau} + \frac{1}{2} B_\mu B_\nu B_0 F_{\rho\sigma} F_{\lambda\tau} - \\
& \qquad - \frac{1}{2} B_\mu B_0 B_\nu F_{\rho\sigma} F_{\lambda\tau} + \frac{1}{2} B_0 B_\mu B_\nu F_{\rho\sigma} F_{\lambda\tau} \Big] - \\
& - \frac{2}{5} g \Big[B_\mu B_\nu F_{\rho\sigma} B_\lambda F_\tau + \frac{1}{2} B_\mu B_\nu F_{\rho\sigma} B_0 F_{\lambda\tau} - B_\mu B_\nu F_\rho B_\sigma F_{\lambda\tau} -
\end{aligned}
$$

[4]The trace is over the adjoint representation of SO(5).

$$
\begin{aligned}
&\qquad - \frac{1}{2} B_\mu B_0 F_{\nu\rho} B_\sigma F_{\lambda\tau} + \frac{1}{2} B_0 B_\mu F_{\nu\rho} B_\sigma F_{\lambda\tau} \Big] + \\
&+ \frac{4}{5} g^2 \Big[B_\mu B_\nu B_\rho B_\sigma B_\lambda F_\tau + \frac{1}{2} B_\mu B_\nu B_\rho B_\sigma B_0 F_{\lambda\tau} - \\
&\qquad - \frac{1}{2} B_\mu B_\nu B_\rho B_0 B_\sigma F_{\lambda\tau} + \frac{1}{2} B_\mu B_\nu B_0 B_\rho B_\sigma F_{\lambda\tau} - \\
&\qquad - \frac{1}{2} B_\mu B_0 B_\nu B_\rho B_\sigma F_{\lambda\tau} + \frac{1}{2} B_0 B_\mu B_\nu B_\rho B_\sigma F_{\lambda\tau} \Big] - \\
&- \frac{4}{5} g^3 [B_\mu B_\nu B_\rho B_\sigma B_\lambda B_\tau B_0] \Big\} \\
-&\frac{1}{2} \epsilon^{\mu\nu\rho\sigma\lambda\tau} \mathrm{Tr}\left(B_\mu F_\nu + \frac{1}{2} B_0 F_{\mu\nu} - g B_0 B_\mu B_\nu \right) \mathrm{Tr}(F_{\rho\sigma} F_{\lambda\tau}) \\
-&\frac{1}{2} \epsilon^{\mu\nu\rho\sigma\lambda\tau} \mathrm{Tr}\left(B_\mu F_{\nu\rho} - \frac{4}{3} g B_\mu B_\nu B_\rho \right) \mathrm{Tr}(F_{\sigma\lambda} F_\tau) \,.
\end{aligned}
\tag{B.1}
$$

C. $D = 6$ maximal supergravity

Here we present the complete lagrangean and supersymmetry transformation laws of D=6 maximal supergravity (i.e. the $g = 0$ limit of the model of appendix A). The field content is 1 graviton $g_{\mu\nu}$, 5 2-index antisymmetric tensor potentials $C_{\mu\nu I}$, (10+5+1) vectors $({B_{\mu I}}^J, S_{\mu I}, A_\mu)$, (14+10+1) scalars $({\Pi_I}^j, {B_{0I}}^J, \sigma)$, 4 gravitini ψ_μ and (16+4) spin 1/2 fermions (λ_i, χ). ψ_μ, λ_i and χ are all D=6 USp(4) symplectic Majorana spinors and the spinor inversion formula reads

$$
\bar{\lambda} \tau^{\alpha_1 \ldots \alpha_n} \gamma^{i_1 \ldots i_r} \chi = (-1)^{\frac{n(n+1)}{2}} (-1)^{\frac{r(r-1)}{2}} \bar{\chi} \tau^{\alpha_1 \ldots \alpha_n} \gamma^{i_1 \ldots i_r} \lambda \tag{C.1}
$$

where γ^i are the D=5 Dirac matrices. The lagrangean of D=6 maximal supergravity, neglecting quartic fermion terms, is:

$$
\begin{aligned}
e^{-1} \mathcal{L}_6 = \; & R - \frac{1}{4} e^{-\frac{5\sigma}{\sqrt{10}}} (f_{\mu\nu})^2 - \frac{1}{12} e^{-\frac{2\sigma}{\sqrt{10}}} \left({{\Pi^{-1}}_i}^I H_{\mu\nu\rho I} \right)^2 - \frac{1}{4} e^{-\frac{\sigma}{\sqrt{10}}} \left({\Pi_I}^i {\Pi_J}^j F^{IJ}_{\mu\nu} \right)^2 - \\
& - \frac{1}{4} e^{\frac{3\sigma}{\sqrt{10}}} \left({{\Pi^{-1}}_i}^I G_{\mu\nu I} \right)^2 - \frac{1}{2} e^{\frac{4\sigma}{\sqrt{10}}} ({\Pi_I}^i {\Pi_J}^j F^{IJ}_\mu)^2 - \frac{1}{2} (\partial_\mu \sigma)^2 - P_{\mu ij} P^{\mu ij} - \\
& - \frac{e^{-1}}{36\sqrt{2}} \epsilon^{\mu\nu\rho\sigma\lambda\tau} {B_0}^{IJ} H_{\mu\nu\rho I} H_{\sigma\lambda\tau J} - \frac{e^{-1}}{6\sqrt{2}} \epsilon^{\mu\nu\rho\sigma\lambda\tau} H_{\mu\nu\rho I} {B_\sigma}^{IJ} G_{\lambda\tau J} - \\
& - \bar{\psi}_\mu \tau^{\mu\nu\rho} \nabla_\nu \psi_\rho - \bar{\chi} \tau^\mu \nabla_\mu \chi - \bar{\lambda}^i \tau^\mu \nabla_\mu \lambda_i + \bar{\psi}_\mu \tau^\nu \tau^\mu \gamma^i \lambda^j P_{\nu ij} + \\
& + \frac{1}{4\sqrt{10}} \bar{\psi}_\mu g^{\mu\nu} \tau^\lambda \psi_\nu \partial_\lambda \sigma - \frac{1}{\sqrt{2}} \bar{\psi}_\mu \tau^\nu \tau^\mu \tau^{\underline{7}} \chi \partial_\nu \sigma - \\
& - \frac{1}{8} \bar{\psi}_\mu [2\tau^{\mu\nu\rho} \tau^\lambda + \tau^{\mu\rho} \tau^{\nu\lambda}] \tau^{\underline{7}} \psi_\rho f_{\nu\lambda} e^{-\frac{5\sigma}{2\sqrt{10}}} - \frac{\sqrt{5}}{8} \bar{\psi}_\mu \tau^{\nu\lambda} \tau^\mu \chi f_{\nu\lambda} e^{-\frac{5\sigma}{2\sqrt{10}}} - \\
& - \frac{3}{16} \bar{\chi} \tau^{\underline{7}} \tau^{\nu\lambda} \chi f_{\nu\lambda} e^{-\frac{5\sigma}{2\sqrt{10}}} - \frac{1}{8} \bar{\lambda}^i \tau^{\underline{7}} \tau^{\nu\lambda} \lambda_i f_{\nu\lambda} e^{-\frac{5\sigma}{2\sqrt{10}}} + \\
& + \frac{1}{8\sqrt{2}} \bar{\psi}_\mu [\tau^{\mu\nu\rho\sigma} - 2g^{\mu\nu} g^{\rho\sigma}] \gamma_{ij} \psi_\sigma {\Pi_I}^i {\Pi_J}^j F^{IJ}_{\nu\rho} e^{-\frac{\sigma}{2\sqrt{10}}} +
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{8\sqrt{10}}\bar{\psi}_\mu[2g^{\mu\nu}\tau^\rho+\tau^{\mu\nu\rho}]\tau^{\underline{7}}\gamma_{ij}\chi\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\nu\rho}e^{-\frac{\sigma}{2\sqrt{10}}}-\\
&-\frac{11}{80\sqrt{2}}\bar{\chi}\tau^{\nu\rho}\gamma_{ij}\chi\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\nu\rho}e^{-\frac{\sigma}{2\sqrt{10}}}+\frac{1}{2\sqrt{2}}\bar{\psi}_\mu\tau^{\nu\rho}\tau^\mu\gamma_i\lambda_j\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\nu\rho}e^{-\frac{\sigma}{2\sqrt{10}}}-\\
&-\frac{1}{2\sqrt{10}}\bar{\chi}\tau^{\nu\rho}\tau^{\underline{7}}\gamma_i\lambda_j\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\nu\rho}e^{-\frac{\sigma}{2\sqrt{10}}}+\\
&+\frac{1}{16\sqrt{2}}\bar{\lambda}_i\gamma^j\gamma_{kl}\gamma^i\tau^{\nu\rho}\lambda_j\Pi_I{}^k\Pi_J{}^l F^{IJ}_{\nu\rho}e^{-\frac{\sigma}{2\sqrt{10}}}-\\
&-\frac{1}{4\sqrt{2}}\bar{\psi}_\mu\tau^{\mu\nu\rho}\tau^{\underline{7}}\gamma_{ij}\psi_\rho\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\nu}e^{\frac{2\sigma}{\sqrt{10}}}-\frac{1}{\sqrt{10}}\bar{\psi}_\mu\tau^\mu\tau^\nu\gamma_{ij}\chi\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\nu}e^{\frac{2\sigma}{\sqrt{10}}}+\\
&+\frac{7}{20\sqrt{2}}\bar{\chi}\tau^{\underline{7}}\tau^\nu\gamma_{ij}\chi\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\nu}e^{\frac{2\sigma}{\sqrt{10}}}-\frac{1}{\sqrt{2}}\bar{\psi}_\mu\tau^\nu\tau^\mu\tau^{\underline{7}}\gamma_i\lambda_j\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\nu}e^{\frac{2\sigma}{\sqrt{10}}}+\\
&+\frac{4}{\sqrt{10}}\bar{\chi}\tau^\nu\gamma_i\lambda_j\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\nu}e^{\frac{2\sigma}{\sqrt{10}}}+\frac{1}{8\sqrt{2}}\bar{\lambda}_i\gamma^j\gamma_{kl}\gamma^i\tau^\nu\tau^{\underline{7}}\lambda_j\Pi_I{}^k\Pi_J{}^l F^{IJ}_{\nu}e^{\frac{2\sigma}{\sqrt{10}}}+\\
&+\frac{i}{8}\bar{\psi}_\mu[\tau^{\mu\nu\rho\lambda}+g^{\mu\nu}g^{\rho\lambda}]\tau^{\underline{7}}\gamma^i\psi_\lambda\Pi^{-1}{}_i{}^I G_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}}+\\
&+\frac{3i}{8\sqrt{5}}\bar{\psi}_\mu[\tau^{\mu\nu\rho}-2g^{\mu\nu}\tau^\rho]\gamma^i\chi\Pi^{-1}{}_i{}^I G_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}}+\\
&+\frac{i}{80}\bar{\chi}\tau^{\underline{7}}\tau^{\nu\rho}\gamma^i\chi\Pi^{-1}{}_i{}^I G_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}}+\frac{i}{4}\bar{\psi}_\mu[\tau^{\mu\nu\rho}-2g^{\mu\nu}\tau^\rho]\tau^{\underline{7}}\lambda^i\Pi^{-1}{}_i{}^I G_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}}-\\
&-\frac{3i}{4\sqrt{5}}\bar{\chi}\tau^{\nu\rho}\lambda^i\Pi^{-1}{}_i{}^I G_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}}+\frac{i}{8}\bar{\lambda}^j\tau^{\nu\rho}\tau^{\underline{7}}\gamma^i\lambda_j\Pi^{-1}{}_i{}^I G_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}}+\\
&+\frac{i}{24}\bar{\psi}_\sigma[\tau^{\sigma\mu\nu\rho\lambda}+6g^{\sigma\mu}\tau^\nu g^{\rho\lambda}]\gamma^i\psi_\lambda\Pi^{-1}{}_i{}^I H_{\mu\nu\rho I}e^{-\frac{\sigma}{\sqrt{10}}}+\\
&+\frac{i}{12\sqrt{5}}\bar{\psi}_\sigma[\tau^{\sigma\mu\nu\rho}-3g^{\sigma\mu}\tau^{\nu\rho}]\tau^{\underline{7}}\gamma^i\chi\Pi^{-1}{}_i{}^I H_{\mu\nu\rho I}e^{-\frac{\sigma}{\sqrt{10}}}+\\
&+\frac{i}{40}\bar{\chi}\tau^{\mu\nu\rho}\gamma^i\chi\Pi^{-1}{}_i{}^I H_{\mu\nu\rho I}e^{-\frac{\sigma}{\sqrt{10}}}-\\
&-\frac{i}{12}\bar{\psi}_\sigma[\tau^{\sigma\mu\nu\rho}-3g^{\sigma\mu}\tau^{\nu\rho}]\lambda^i\Pi^{-1}{}_i{}^I H_{\mu\nu\rho I}e^{-\frac{\sigma}{\sqrt{10}}}-\\
&-\frac{i}{6\sqrt{5}}\bar{\chi}\tau^{\underline{7}}\tau^{\mu\nu\rho}\lambda^i\Pi^{-1}{}_i{}^I H_{\mu\nu\rho I}e^{-\frac{\sigma}{\sqrt{10}}}-\frac{i}{24}\bar{\lambda}^j\tau^{\mu\nu\rho}\gamma^i\lambda_j\Pi^{-1}{}_i{}^I H_{\mu\nu\rho I}e^{-\frac{\sigma}{\sqrt{10}}}\,, \qquad \text{(C.2)}
\end{aligned}
$$

where

$$
\begin{aligned}
f_2 &= dA\,, \qquad\qquad G_{2I} = dS_{1I}\,,\\
F_{2I}{}^J &= dB_{1I}{}^J + B_{0I}{}^J dA\,, \qquad F_{1I}{}^J = dB_{0I}{}^J\,,\\
H_{\mu\nu\rho I} &= 3\left(\partial_{[\mu}C_{\nu\rho]I}+\frac{1}{2}G_{[\mu\nu I}A_{\rho]}\right),\\
P_{\mu ij} &= \Pi^{-1}{}_{(i}^{\;I}\partial_\mu\Pi_{Ij)}\,,\\
Q_{\mu ij} &= \Pi^{-1}{}_{[i}^{\;I}\partial_\mu\Pi_{Ij]}
\end{aligned}
\qquad \text{(C.3)}
$$

and

$$
\nabla_\mu\xi_{\nu i} = \partial_\mu\xi_{\nu i}+\frac{1}{4}\omega_{\mu\underline{\nu\rho}}\tau^{\underline{\nu\rho}}\xi_{\nu i}+\Gamma^\rho_{\mu\nu}\xi_{\rho i}+\frac{1}{4}Q_{\mu jk}\gamma^{jk}\xi_{\nu i}+Q_{\mu i}{}^j\xi_{\nu j}, \qquad \text{(C.4)}
$$

for a general vector spinor, $\xi_{\nu i}$, of both SO(5,1) and SO(5)$_c$. ψ_μ and χ are SO(5)$_c$ spinors and λ_i is an SO(5)$_c$ vector spinor. The supersymmetry transformations of the fields neglecting terms cubic in fermion fields are:

$$\begin{aligned}\delta\psi_\mu = {} & \nabla_\mu\epsilon - \frac{1}{32}[\tau_\mu{}^{\nu\rho} - 6\delta^\nu_\mu\tau^\rho]\tau^{\underline{7}}\epsilon f_{\nu\rho}e^{-\frac{5\sigma}{2\sqrt{10}}} - \\ & -\frac{1}{32\sqrt{2}}[\tau_\mu{}^{\nu\rho} - 6\delta^\nu_\mu\tau^\rho]\gamma_{ij}\epsilon\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\nu\rho}e^{-\frac{\sigma}{2\sqrt{10}}} + \frac{1}{4\sqrt{2}}\tau^{\underline{7}}\gamma_{ij}\epsilon\Pi_I{}^i\Pi_J{}^j F^{IJ}_\mu e^{\frac{2\sigma}{\sqrt{10}}} + \\ & +\frac{1}{2\sqrt{10}}\epsilon\partial_\mu\sigma - \frac{i}{32}[\tau_\mu{}^{\nu\rho} - 6\delta^\nu_\mu\tau^\rho]\tau^{\underline{7}}\gamma^i\epsilon\Pi^{-1}{}_i{}^I G_{\nu\rho I}e^{\frac{3\sigma}{2\sqrt{10}}} + \\ & +\frac{i}{48}[\tau_\mu{}^{\nu\rho\sigma} - 3\delta^\nu_\mu\tau^{\rho\sigma}]\gamma^i\epsilon\Pi^{-1}{}_i{}^I H_{\nu\rho\sigma I}e^{-\frac{\sigma}{\sqrt{10}}}\,,\end{aligned} \tag{C.5}$$

$$\begin{aligned}\delta\chi = {} & -\frac{\sqrt{5}}{16}\tau^{\mu\nu}\epsilon f_{\mu\nu}e^{-\frac{5\sigma}{2\sqrt{10}}} + \frac{1}{2\sqrt{2}}\tau^\mu\tau^{\underline{7}}\epsilon\partial_\mu\sigma - \frac{1}{16\sqrt{10}}\tau^{\mu\nu}\tau^{\underline{7}}\gamma_{ij}\epsilon\Pi_I{}^i\Pi_J{}^j F^{IJ}_{\mu\nu}e^{-\frac{\sigma}{2\sqrt{10}}} - \\ & -\frac{1}{2\sqrt{10}}\tau^\mu\gamma_{ij}\epsilon\Pi_I{}^i\Pi_J{}^j F^{IJ}_\mu e^{\frac{2\sigma}{\sqrt{10}}} + \frac{3i}{16\sqrt{5}}\tau^{\mu\nu}\gamma^i\epsilon\Pi^{-1}{}_i{}^I G_{\mu\nu I}e^{\frac{3\sigma}{2\sqrt{10}}} - \\ & -\frac{i}{24\sqrt{5}}\tau^{\mu\nu\rho}\tau^{\underline{7}}\gamma^i\epsilon\Pi^{-1}{}_i{}^I H_{\mu\nu\rho I}e^{-\frac{\sigma}{\sqrt{10}}}\,,\end{aligned} \tag{C.6}$$

$$\begin{aligned}\delta\lambda_i = {} & \frac{1}{16\sqrt{2}}\tau^{\mu\nu}[\gamma_{kl}\gamma_i - \frac{1}{5}\gamma_i\gamma_{kl}]\epsilon\Pi_I{}^k\Pi_J{}^l F^{IJ}_{\mu\nu}e^{-\frac{\sigma}{2\sqrt{10}}} + \frac{1}{2}\tau^\mu\gamma^j\epsilon P_{\mu ij} + \\ & +\frac{1}{8\sqrt{2}}\tau^\mu\tau^{\underline{7}}[\gamma_{kl}\gamma_i - \frac{1}{5}\gamma_i\gamma_{kl}]\epsilon\Pi_I{}^k\Pi_J{}^l F^{IJ}_\mu e^{\frac{2\sigma}{\sqrt{10}}} - \\ & -\frac{i}{40}\tau^{\mu\nu}\tau^{\underline{7}}[\gamma_i{}^j - 4\delta_i^j]\epsilon\Pi^{-1}{}_j{}^I G_{\mu\nu I}e^{\frac{3\sigma}{2\sqrt{10}}} + \\ & +\frac{i}{120}\tau^{\mu\nu\rho}[\gamma_i{}^j - 4\delta_i^j]\epsilon\Pi^{-1}{}_j{}^I H_{\mu\nu\rho I}e^{-\frac{\sigma}{\sqrt{10}}}\,,\end{aligned} \tag{C.7}$$

$$\delta\sigma = -\frac{1}{\sqrt{2}}\bar{\epsilon}\tau^{\underline{7}}\chi e^{\frac{\sigma}{\sqrt{10}}}\,, \qquad \Pi^{-1}{}_i{}^I\delta\Pi_I{}^j = \frac{1}{4}[\bar{\epsilon}\gamma_i\lambda^j + \bar{\epsilon}\gamma^j\lambda_i]e^{\frac{\sigma}{\sqrt{10}}}\,, \tag{C.8}$$

$$\delta A_\mu = \frac{1}{2}\bar{\epsilon}\tau^{\underline{7}}\psi_\mu e^{\frac{7\sigma}{2\sqrt{10}}} + \frac{1}{4\sqrt{5}}\bar{\epsilon}\tau_\mu\chi e^{\frac{7\sigma}{2\sqrt{10}}}\,, \tag{C.9}$$

$$\delta e^{\underline{\mu}}{}_\mu = \frac{1}{2}\bar{\epsilon}\tau^{\underline{\mu}}\psi_\mu e^{\frac{\sigma}{\sqrt{10}}} + \frac{1}{\sqrt{5}}\bar{\epsilon}\tau^{\underline{\mu}}\chi A_\mu e^{-\frac{3\sigma}{2\sqrt{10}}} + \frac{1}{4\sqrt{5}}\bar{\epsilon}\tau_\mu{}^\nu e^{\underline{\mu}}{}_\nu\tau^{\underline{7}}\chi e^{\frac{\sigma}{\sqrt{10}}}\,, \tag{C.10}$$

$$\begin{aligned}\Pi_I{}^i\Pi_J{}^j\delta B_0{}^{IJ} = {} & \frac{1}{\sqrt{10}}\bar{\epsilon}\gamma^{ij}\chi e^{-\frac{\sigma}{\sqrt{10}}} - \frac{1}{\sqrt{5}}\bar{\epsilon}\tau^\mu\chi\Pi_I{}^i\Pi_J{}^j B_\mu{}^{IJ}e^{-\frac{3\sigma}{2\sqrt{10}}} + \\ & +\frac{1}{4\sqrt{2}}\bar{\epsilon}\tau^{\underline{7}}\gamma^k\gamma^{ij}\lambda_k e^{-\frac{\sigma}{\sqrt{10}}},\end{aligned} \tag{C.11}$$

$$\Pi_I{}^i\Pi_J{}^j\delta B_\mu{}^{IJ} = \frac{1}{2\sqrt{2}}\bar\epsilon\gamma^{ij}\psi_\mu e^{\frac{3\sigma}{2\sqrt{10}}} + \frac{1}{4\sqrt{10}}\bar\epsilon\gamma^{ij}\tau^{\underline{7}}\tau_\mu\chi e^{\frac{3\sigma}{2\sqrt{10}}} + \frac{1}{4\sqrt{2}}\bar\epsilon\tau_\mu\gamma^k\gamma^{ij}\lambda_k e^{\frac{3\sigma}{2\sqrt{10}}} -$$
$$-\frac{1}{2}\bar\epsilon\tau^{\underline{7}}\psi_\mu\Pi_I{}^i\Pi_J{}^j B_0{}^{IJ}e^{\frac{7\sigma}{2\sqrt{10}}} - \frac{1}{4\sqrt{5}}\bar\epsilon\tau_\mu\chi\Pi_I{}^i\Pi_J{}^j B_0{}^{IJ}e^{\frac{7\sigma}{2\sqrt{10}}} +$$
$$+\frac{1}{\sqrt{5}}\bar\epsilon\tau^\nu A_\mu\chi\Pi_I{}^i\Pi_J{}^j B_\nu{}^{IJ}e^{-\frac{3\sigma}{2\sqrt{10}}}\,, \tag{C.12}$$

$$\delta G_{\mu\nu I} = -\frac{2}{\sqrt{5}}\bar\epsilon\tau^\rho\chi A_\mu G_{\nu\rho I}e^{-\frac{3\sigma}{2\sqrt{10}}} +$$
$$+\frac{i}{4\sqrt{2}}\Pi_I{}^i\left[\frac{4}{\sqrt{5}}\bar\epsilon\gamma_{ijk}\chi + \bar\epsilon\tau^{\underline{7}}\gamma^l\gamma_{ijk}\lambda_l\right]\Pi_J{}^j\Pi_K{}^k F^{JK}_{\mu\nu}e^{-\frac{\sigma}{\sqrt{10}}} +$$
$$+\frac{i}{4\sqrt{2}}\Pi_I{}^i\left[4\bar\epsilon\gamma_{ijk}\psi_\mu + \frac{2}{\sqrt{5}}\bar\epsilon\gamma_{ijk}\tau^{\underline{7}}\tau_\mu\chi + 2\bar\epsilon\tau_\mu\gamma^l\gamma_{ijk}\lambda_l\right]\Pi_J{}^j\Pi_K{}^k F^{JK}_\nu e^{\frac{3\sigma}{2\sqrt{10}}} +$$
$$+\frac{1}{\sqrt{5}}\bar\epsilon\tau^\rho\chi H_{\mu\nu\rho I}e^{-\frac{3\sigma}{2\sqrt{10}}}\,, \tag{C.13}$$

$$\delta H_{\mu\nu\rho I} = \frac{3}{2}\bar\epsilon\tau^{\underline{7}}\psi_\mu G_{\nu\rho I}e^{\frac{7\sigma}{2\sqrt{10}}} + \frac{3}{4\sqrt{5}}\bar\epsilon\tau_\mu\chi G_{\nu\rho I}e^{\frac{7\sigma}{2\sqrt{10}}} + \frac{3}{\sqrt{5}}\bar\epsilon\tau^\lambda\chi A_\mu H_{\nu\rho\lambda I}e^{-\frac{3\sigma}{2\sqrt{10}}} -$$
$$-\frac{3i}{4\sqrt{2}}\Pi_I{}^i\left[2\bar\epsilon\gamma_{ijk}\psi_\mu + \frac{1}{\sqrt{5}}\bar\epsilon\gamma_{ijk}\tau^{\underline{7}}\tau_\mu\chi + \bar\epsilon\tau_\mu\gamma^l\gamma_{ijk}\lambda_l\right]\Pi_J{}^j\Pi_K{}^k F^{JK}_{\nu\rho}e^{\frac{3\sigma}{2\sqrt{10}}}\,. \tag{C.14}$$

D. $D = 5$ $N = 4$ SU(2) $\times$ U(1) supergravity in the global limit

The bosonic sector of the D=5 N=4 SU(2) gauged supergravity was obtained in [17] by reduction on T^2 of D=7 SU(2) gauged supergravity [18]. The bosonic sector of the lagrangean is [17]:

$$e^{-1}\mathcal{L} = R_5 - \frac{1}{2}|d\psi|^2 - \frac{1}{4}e^{-\frac{2\psi}{\sqrt{6}}}\left[|F_2|^2 + |G_2^{(1)}|^2 + |G_2^{(2)}|^2\right] - \frac{1}{4}e^{\frac{4\psi}{\sqrt{6}}}|C_2|^2 + 4\alpha^2 e^{\frac{2\psi}{\sqrt{6}}} -$$
$$-\frac{e^{-1}}{8}\epsilon^{\mu\nu\rho\sigma\tau}C_\mu\left[G^{(1)}_{\nu\rho}G^{(1)}_{\sigma\tau} + G^{(2)}_{\nu\rho}G^{(2)}_{\sigma\tau} + \mathrm{Tr}(F_{\nu\rho}F_{\sigma\tau})\right] \tag{D.1}$$

where $G_2^{(p)} = dB_1^{(p)}$, (p=1,2), $C_2 = dC_1$ and $F_{\mu\nu i}{}^j = 2(\partial_{[\mu}A_{\nu]i}{}^j + i\alpha A_{[\mu|i|}{}^k A_{\nu]k}{}^j)$, ($i$,$j$=1,2). Hence the global SO(2) symmetry is manifest. The SU(2)$\times$U(1) gauged model of Romans [16] contains a pair of 2-index potentials $B^\alpha_{\mu\nu}$ instead of the vectors $B^{(p)}_\mu$ in (D.1). This allows the U(1) symmetry to be gauged. The bosonic sector of Romans' model is the same as (D.1) except there is an extra term in the scalar potential proportional to the U(1) coupling constant g_1 and instead of the terms

$$-\frac{e}{4}\xi^2|G^{(1)}_{\mu\nu}|^2 - \frac{e}{4}\xi^2|G^{(2)}_{\mu\nu}|^2 - \frac{1}{8}\epsilon^{\mu\nu\rho\sigma\tau}C_\mu\left[G^{(1)}_{\nu\rho}G^{(1)}_{\sigma\tau} + G^{(2)}_{\nu\rho}G^{(2)}_{\sigma\tau}\right], \tag{D.2}$$

where $\xi = e^{-\frac{\psi}{\sqrt{6}}}$, Romans' model contains the terms

$$\mathcal{L} = \epsilon^{\mu\nu\rho\sigma\tau}\left[\frac{1}{g_1}\epsilon_{\alpha\beta}B^{\alpha}_{\mu\nu}D_\rho B^{\beta}_{\sigma\tau}\right] - e\xi^2 B^{\mu\nu\alpha}B_{\mu\nu\alpha}\,, \tag{D.3}$$

where α=1,2 and $D_\mu B^{\alpha}_{\nu\rho} = \partial_\mu B^{\alpha}_{\nu\rho} + \frac{g_1}{2}\epsilon^{\alpha\beta}C_\mu B_{\nu\rho\beta}$. We now show that the U(1) coupling constant g_1 *can* be taken to zero[5] and the lagrangean (D.3) in this limit becomes (D.2), thus showing the SU(2) gauged model (D.1) is indeed the limit of Romans' SU(2)×U(1) gauged supergravity in which the U(1) gauging is turned off.

Variation of (D.3) w.r.t. $B^1_{\mu\nu}$ yields the field equation

$$e\xi^2 B^{\mu\nu 1} = \frac{1}{g_1}\epsilon^{\mu\nu\rho\sigma\tau}\left[\partial_\rho B^2_{\sigma\tau} - \frac{g_1}{2}C_\rho B^1_{\sigma\tau}\right]. \tag{D.4}$$

Hence solving for $B^1_{\mu\nu}$ we have

$$P^{\mu\nu\sigma\tau}B^1_{\sigma\tau} = \frac{1}{3g_1}\epsilon^{\mu\nu\rho\sigma\tau}H_{\rho\sigma\tau}\,, \tag{D.5}$$

where $H_{\mu\nu\rho} = 3\partial_\mu B^2_{\nu\rho}$ and we have defined the operator P s.t.

$$P^{\mu\nu\rho\sigma} = e\xi^2 g^{\mu\rho}g^{\nu\sigma} + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma\tau}C_\tau\,. \tag{D.6}$$

Hence we see $B^1_{\mu\nu}$ is an auxiliary field which we can eliminate. Defining the inverse of P s.t.

$$P^{\mu\nu\rho\sigma}(P^{-1})_{\rho\sigma\alpha\beta} = \delta^{\mu\nu}_{\alpha\beta}, \tag{D.7}$$

we can solve (D.5) for $B^1_{\mu\nu}$ and substitute back in the lagrangean (D.3). Hence we obtain a lagrangean involving just the field $B^2_{\mu\nu}$:

$$\mathcal{L} = \frac{1}{9g_1}[\epsilon^{\mu\nu\rho\sigma\tau}H_{\rho\sigma\tau}](P^{-1})_{\mu\nu\alpha\beta}[\epsilon^{\alpha\beta\gamma\delta\epsilon}H_{\gamma\delta\epsilon}] - B^2_{\mu\nu}P^{\mu\nu\sigma\tau}B^2_{\sigma\tau}\,. \tag{D.8}$$

Before we take g_1 to zero we must regurgitate a vector from $B^2_{\mu\nu}$ thus

$$B^2_{\mu\nu} \longrightarrow B^2_{\mu\nu} + \frac{1}{g_1}G^{(2)}_{\mu\nu}\,, \tag{D.9}$$

where $G^{(2)}_{\mu\nu} = 2\partial_\mu B^{(2)}_\nu$. The lagrangean therefore becomes:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{{g_1}^2}G^{(2)}_{\mu\nu}P^{\mu\nu\rho\sigma}G^{(2)}_{\rho\sigma} - \frac{2}{g_1}G^{(2)}_{\mu\nu}P^{\mu\nu\rho\sigma}B^2_{\rho\sigma} - B^2_{\mu\nu}P^{\mu\nu\rho\sigma}B^2_{\rho\sigma} + \\ &+\frac{1}{9g_1}[\epsilon^{\mu\nu\rho\sigma\tau}H_{\rho\sigma\tau}](P^{-1})_{\mu\nu\alpha\beta}[\epsilon^{\alpha\beta\gamma\delta\epsilon}H_{\gamma\delta\epsilon}]\,.\end{aligned} \tag{D.10}$$

[5]contrary to the statement that g_1 cannot be taken to zero in [16].

Now we see that after making the following field rescalings $B^2_{\mu\nu} \longrightarrow \sqrt{g_1}B^2_{\mu\nu}$, $G^{(2)}_{\mu\nu} \longrightarrow g_1 G^{(2)}_{\mu\nu}$, the g_1=0 limit can be obtained:

$$\mathcal{L} = -G^{(2)}_{\mu\nu}P^{\mu\nu\rho\sigma}G^{(2)}_{\rho\sigma} + \frac{1}{9}[\epsilon^{\mu\nu\rho\sigma\tau}H_{\rho\sigma\tau}](P^{-1})_{\mu\nu\alpha\beta}[\epsilon^{\alpha\beta\gamma\delta\epsilon}H_{\gamma\delta\epsilon}]\,. \tag{D.11}$$

The extra term in the scalar potential proportional to g_1 simply vanishes in this limit.

In order to compare with the model (D.2) (which contains 2 massless vectors) we must dualise $H_{\gamma\delta\epsilon}$ to $G^{(1)}_{\mu\nu}$. This is achieved by replacing $3\partial_\mu B^2_{\nu\rho}$ by an independent field $a_{\mu\nu\rho}$ and adding to $\mathcal{L}$ the term

$$\Delta\mathcal{L} = \kappa\epsilon^{\mu\nu\rho\sigma\tau}a_{\mu\nu\rho}G^{(1)}_{\sigma\tau}\,, \tag{D.12}$$

where κ is a constant and $G^{(1)}_{\sigma\tau} = 2\partial_\sigma B^{(1)}_\tau$. Variation of $\mathcal{L}+\Delta\mathcal{L}$ w.r.t. $a_{\mu\nu\rho}$ then gives

$$\frac{2}{9}(P^{-1})_{\alpha\beta\mu\nu}[\epsilon^{\mu\nu\rho\sigma\tau}a_{\rho\sigma\tau}] = -\kappa G^{(1)}_{\alpha\beta}\,. \tag{D.13}$$

Substituting back in $\mathcal{L}+\Delta\mathcal{L}$ (choosing $\kappa = 1/3$ and rescaling $G^{(2)}_2$) gives:

$$\mathcal{L}+\Delta\mathcal{L} = -\frac{1}{4}G^{(1)}_{\mu\nu}P^{\mu\nu\rho\sigma}G^{(1)}_{\rho\sigma} - \frac{1}{4}G^{(2)}_{\mu\nu}P^{\mu\nu\rho\sigma}G^{(2)}_{\rho\sigma}\,, \tag{D.14}$$

thus using the expression for $P^{\mu\nu\rho\sigma}$, (D.6), the lagrangean becomes identical to (D.2). Hence we have shown that the limit of the bosonic sector of D=5 N=4 SU(2)×U(1) gauged supergravity, in which the U(1) coupling constant is turned off, is the D=5 N=4 SU(2) gauged model (D.1) obtained in [17].

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